

# Erratum to “Opportunistic UAV Navigation With Carrier Phase Measurements From Asynchronous Cellular Signals”

**JOE KHALIFE**, Student Member, IEEE  
**ZAHER M. KASSAS**, Senior Member, IEEE  
University of California, Irvine, CA, USA



**Joe Khalife** (Student Member, IEEE) received the B.E. degree in electrical engineering and the M.S. degree in computer engineering from the Lebanese American University (LAU), Beirut, Lebanon, in 2011 and 2014 respectively. He is currently working toward the Ph.D. degree in electrical engineering with the University of California, Irvine, USA.

From 2012 to 2015, he was a Research Assistant with LAU. He is a member of the Autonomous Systems Perception, Intelligence, and Navigation (ASPIN) Laboratory, Irvine, CA, USA. His research interests include opportunistic navigation, autonomous vehicles, and software-defined radio. He was the recipient of the 2018 IEEE Walter Fried Award.

**Zaher (Zak) M. Kassas** (Senior Member, IEEE) received the B.E. degree in electrical engineering from Lebanese American University (LAU), Beirut, Lebanon, the M.S. degree in electrical and computer engineering from The Ohio State University, Columbus, USA, and the M.S.E. degree in aerospace engineering and the Ph.D. degree in electrical and computer engineering from The University of Texas, Austin, USA.

He is an Assistant Professor with the University of California, Irvine, USA, and Director with the ASPIN Laboratory, Irvine, CA, USA. He is an Associate Editor for the IEEE TRANSACTIONS ON AEROSPACE AND ELECTRONIC SYSTEMS and the IEEE TRANSACTIONS ON INTELLIGENT TRANSPORTATION SYSTEMS. His research interests include cyber-physical systems, estimation theory, navigation systems, autonomous vehicles, and intelligent transportation systems.

Dr. Kassas was the recipient of the National Science Foundation (NSF) Faculty Early Career Development Program (CAREER) Award in 2018 and the Office of Naval Research (ONR) Young Investigator Program (YIP) Award in 2019. He was the recipient of 2018 IEEE Walter Fried Award, the 2018 Institute of Navigation (ION) Samuel Burka Award, and the 2019 ION Col. Thomas Thurlow Award.

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Authors' addresses: Joe Khalife is with the Department of Electrical Engineering and Computer Science, University of California Irvine, CA 92617 USA, E-mail: (khalifej@uci.edu); Zaher M. Kassas is with the Department of Mechanical and Aerospace Engineering and the Department of Electrical Engineering and Computer Science, University of California Irvine, CA 92617 USA, E-mail: (zkassas@iee.org). (Corresponding author: Zaher M. Kassas.)

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## REFERENCE

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## Correspondence

This article presents a framework for opportunistic unmanned aerial vehicle (UAV) navigation by exploiting carrier phase measurements from ambient cellular signals of opportunity. In the proposed framework, the cellular base transceiver stations (BTSSs) are not assumed to be synchronous. A complete framework that employs an extended Kalman filter (EKF) is presented, including filter initialization and process and measurement noise covariance selection. The EKF estimates the position and velocity of the UAV, as well as the differences between the UAV-mounted receiver and each of the BTSSs' clock bias and clock drift. The observability of the estimation framework is analyzed, and the boundedness of the EKF's errors is studied. It is shown that the system is observable given a class of vehicle and receiver clock dynamics. A lower bound for the EKF estimation error covariance is derived, and it is shown that the covariance remains bounded. Monte Carlo simulations are conducted to study the effect of the number of BTSSs, the initial UAV speed, and the receiver's oscillator quality, on the estimation performance. Two sets of experimental results are presented demonstrating UAVs navigating exclusively with cellular carrier phase measurements via the developed framework, achieving a total position root-mean-squared error of 2.94 and 5.99 m for UAV trajectories of 2.6 and 2.9 km, respectively.

### I. INTRODUCTION

Current unmanned aerial vehicle (UAV) navigation systems will not meet the stringent requirements on accuracy, resiliency, and robustness due to their heavy reliance on jamnable and spoofable global navigation satellite system (GNSS) signals [1], [2]. In addition to jamming and spoofing, unintentional interference or even the possible yet unlikely event of on-board GNSS receiver failure constitute major threats on safe and reliable UAV navigation. In such cases where GNSS signals are unusable, cellular signals of opportunity (SOPs) could be used for navigation either in a standalone fashion [3], [4] or an integrated fashion, aiding the UAV's inertial navigation system [5],

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Authors' addresses: J. Khalife is with the Department of Electrical Engineering and Computer Science, University of California, Irvine, CA 92617 USA, E-mail: (khalifej@uci.edu); Z. M. Kassas is with the Department of Mechanical and Aerospace Engineering and the Department of Electrical Engineering and Computer Science, University of California, Irvine, CA 92617 USA, E-mail: (zkassas@ieee.org). (*Corresponding author: Zaher M. Kassas.*)

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[6]. Several receiver designs have been published recently, producing time-of-arrival (TOA) and frequency-of-arrival (FOA) measurements from cellular code-division multiple access (CDMA) and long-term evolution (LTE) signals [7]–[10]. Such measurements are typically used in navigation systems. Cellular signals are attractive for navigation, since they are abundant, received at a much higher power than GNSS signals, possess a favorable horizontal geometry, and are free to use. Moreover, cellular signals received by UAVs do not suffer from severe multipath by virtue of the favorable channel between base stations and UAVs [11]. In fact, a recent study of UAV connectivity to the cellular network demonstrated that the received cellular signal power on low-altitude UAVs (30–120 m) is stronger than the receiver power on ground-based receivers, despite the downward-tilted cellular antennas [12]. The study attributes these findings to the fact that “free space propagation conditions at altitude more than make up for antenna gain reductions.”

While TOA-based and FOA-based navigation approaches are well studied in the literature [13], [14], applying such approaches to cellular CDMA base transceiver stations (BTSs) or LTE eNodeBs requires perfect synchronization assumptions [15], [16]. However, cellular CDMA and LTE networks are not perfectly synchronized, and their protocols recommend synchronization of CDMA BTSs and LTE eNodeBs to within  $3 \mu\text{s}$  from GPS time [17], [18]. This translates to ranging errors of about 900 m. Several approaches in the literature have been proposed to account for the BTSs’ or eNodeBs’ clock biases and drifts, including using the round-trip time (RTT) instead of the TOA [19]. Although RTT-based methods could yield good results in asynchronous systems, two-way communication between the receiver and the BTSs or eNodeBs is needed. This limits the availability of RTT measurements to only paying subscribers to a particular cellular provider and compromises the privacy of the user. Some of the proposed navigation frameworks assume the BTSs’ or eNodeBs’ clock bias and drift to be constant [3], [20]. However, the clock bias and drift are dynamic and stochastic [21] and, hence, must be continuously estimated.

To deal with this challenge, a framework employing a monitor receiver was put forth by Carter *et al.* [22]. Moreover, a mapper/navigator framework was proposed in [23], where the mapper, which was assumed to have complete knowledge of its states (e.g., by having access to GNSS signals), is estimating the clock states of BTSs in its environment, and is sharing these estimates with a navigating receiver that has no knowledge of its own states, but is making pseudorange measurements on the same BTSs in the environment. The mapper/navigator framework could yield centimeter-accurate UAV navigation when carrier phase observables extracted from cellular signals are exploited [24]. Having a mapper may be impractical in some environments or in the absence of a communication channel between the mapper and navigator. To alleviate the need of a monitor or a mapper in the case of code

phase measurements from cellular signals, the navigator could estimate its states simultaneously with the states (position, clock bias, and clock drift) of the BTSs in the environment, i.e., perform radio simultaneous localization and mapping [5], [25], [26]. Alternatively, in the case where the navigating UAV is making carrier phase measurements from cellular signals, the relative frequency stability of cellular CDMA BTSs or LTE eNodeBs, which approaches that of an atomic standard, may be leveraged. It was shown that sophisticated measurement models that capture this stability may be employed to achieve centimeter-accurate navigation solutions without a mapper [24]. However, this method may fail if the frequency stability requirement is not met.

This article considers UAV navigation with cellular carrier phase measurements without any assumptions on the synchronization between cellular BTSs or eNodeBs nor on their relative frequency stability. This article makes the following five contributions. First, a framework for UAV navigation with carrier phase measurements from cellular SOPs is presented, which employs an extended Kalman filter (EKF). The precision of carrier phase measurements is on the order of the carrier signal wavelength, making such measurements attractive for UAV navigation. Second, the EKF initialization is discussed. While the stability of the Kalman filter (KF) has been well studied for linear systems, stability of the EKF is carried out on a case-by-case basis. To this end, the third contribution of this article is a complete EKF estimation error and estimation error covariance analysis, performed by studying the observability of the system under consideration as well as the EKF’s stochastic stability. The theoretical results produced herein can be generalized to a broader class of problems: EKF-based navigation using SOP carrier phase and pseudorange measurements. It is shown that the system is observable given a class of vehicle and receiver clock dynamics, and that the EKF estimation error is statistically bounded. Fourth, Monte Carlo (MC) simulations are conducted to demonstrate the theoretical predictions about the system and study the effect of the number of available BTSs and/or eNodeBs available, the initial UAV speed, and the UAV-mounted receiver’s clock quality on the estimation performance. Fifth, two sets of experimental results are presented demonstrating UAVs navigating with the proposed framework achieving a root-mean-squared error (RMSE) of 2.94 and 5.99 m for UAV trajectories of 2.6 and 2.9 km, respectively.

The remainder of this article is organized as follows. Section II describes the cellular SOP and receiver dynamics models and the cellular carrier phase observable. Section III describes the EKF-based navigation framework. Section IV analyzes the observability and the EKF error boundedness of the proposed framework. Section V characterizes the performance of the proposed framework via MC simulations. Section VI provides experimental results showing meter-level UAV navigation accuracy. Section VII concludes this article.

## II. MODEL DESCRIPTION

This section presents the dynamics model of the UAV-mounted receiver and cellular SOP, as well as the cellular carrier phase measurement model. Note that an altimeter could be used to estimate the UAV's altitude. Therefore, only the UAV's two-dimensional (2-D) position is estimated in this article. The subsequent analysis is readily extendable to three-dimensional (3-D); however, the vertical position estimate will suffer from large uncertainty due to the poor vertical diversity of cellular towers.

### A. Cellular SOP Dynamics Model

The cellular SOPs emanate from spatially stationary terrestrial BTSs or eNodeBs, and their states will consist of their *known* 2-D positions and *unknown* clock error states, namely the clock bias and clock drift. For simplicity of notation, let the term BTS denote any type of cellular SOP transmitter (CDMA BTS, LTE eNodeB, etc.), unless explicitly stated otherwise. The position vector of the  $n$ th BTS is given by  $\mathbf{r}_{s_n} = [x_{s_n}, y_{s_n}]^T$ . The state of the  $n$ th BTS will only consist of its clock error state and is given by  $\mathbf{x}_{\text{clk},s_n}(k) \triangleq [c\delta t_{s_n}(k), c\dot{\delta}t_{s_n}(k)]^T$ , where  $\delta t_{s_n}(k)$  and  $\dot{\delta}t_{s_n}(k)$  are the clock bias and clock drift at time step  $k$ , respectively, and  $c$  is the speed of light. The  $n$ th BTS's dynamics can be described by the discretized state-space model

$$\mathbf{x}_{\text{clk},s_n}(k+1) = \mathbf{F}_{\text{clk}} \mathbf{x}_{\text{clk},s_n}(k) + \mathbf{w}_{\text{clk},s_n}(k), \quad k = 0, 1, \dots$$

where  $n = 1, \dots, N$ , with  $N$  being the total number of BTSs, and  $\mathbf{w}_{\text{clk},s_n}(k)$  is a zero-mean white noise sequence with covariance  $\mathbf{Q}_{\text{clk},s_n}$ , with

$$\mathbf{F}_{\text{clk}} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad \mathbf{Q}_{\text{clk},s_n} = c^2 \begin{bmatrix} S_{\tilde{w}_{\delta t_{s_n}}} T + S_{\tilde{w}_{\dot{\delta}t_{s_n}}} \frac{T^3}{3} & S_{\tilde{w}_{\delta t_{s_n}}} \frac{T^2}{2} \\ S_{\tilde{w}_{\dot{\delta}t_{s_n}}} \frac{T^2}{2} & S_{\tilde{w}_{\delta t_{s_n}}} T \end{bmatrix}$$

where  $T$  is the sampling time, and  $S_{\tilde{w}_{\delta t_{s_n}}}$  and  $S_{\tilde{w}_{\dot{\delta}t_{s_n}}}$  are the power spectra of the continuous-time process noise driving the clock bias and clock drift, respectively. These spectra can be related to the power law coefficients  $\{h_\alpha\}_{\alpha=-2}^2$ , which have been shown through laboratory experiments to be adequate to characterize the power spectral density of the fractional frequency deviation  $y(t)$  of an oscillator from nominal frequency, which takes the form  $S_y(f) = \sum_{\alpha=-2}^2 h_\alpha f^\alpha$  [27]. It is common to approximate the clock error dynamics by considering only the frequency random walk coefficient  $h_{-2}$  and the white frequency coefficient  $h_0$ , which leads to  $S_{\tilde{w}_{\delta t_{s_n}}} \approx \frac{h_{0,s_n}}{2}$  and  $S_{\tilde{w}_{\dot{\delta}t_{s_n}}} \approx 2\pi^2 h_{-2,s_n}$  [28].

### B. UAV-Mounted Receiver Dynamics Model

The UAV-mounted receiver state consists of its unknown position  $\mathbf{r}_r(k) \triangleq [x_r(k), y_r(k)]^T$ , velocity  $\dot{\mathbf{r}}_r(k)$ , and clock error states  $\mathbf{x}_{\text{clk},r}(k) \triangleq [c\delta t_r(k), c\dot{\delta}t_r(k)]^T$ . Hence, the state vector of the receiver is given by  $\mathbf{x}_r(k) = [\mathbf{r}_r^T(k), \dot{\mathbf{r}}_r^T(k), \mathbf{x}_{\text{clk},r}^T(k)]^T$ . The receiver's position  $\mathbf{r}_r(k)$  and velocity  $\dot{\mathbf{r}}_r(k)$  will be assumed to evolve according to a

velocity random walk model [29]. Therefore, the UAV-mounted receiver dynamics is modeled according to the discretized model

$$\mathbf{x}_r(k+1) = \mathbf{F}_r \mathbf{x}_r(k) + \mathbf{w}_r(k), \quad k = 0, 1, \dots$$

where  $\mathbf{w}_r(k) = [\mathbf{w}_{\text{pv}}^T(k), \mathbf{w}_{\text{clk},r}^T(k)]^T$  is a discrete-time zero-mean white noise sequence with covariance  $\mathbf{Q}_r = \text{diag}[\mathbf{Q}_{\text{pv}}, \mathbf{Q}_{\text{clk},r}]$ , with

$$\mathbf{F}_r = \begin{bmatrix} \mathbf{I}_{2 \times 2} & T\mathbf{I}_{2 \times 2} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & \mathbf{I}_{2 \times 2} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} & \mathbf{F}_{\text{clk}} \end{bmatrix}, \quad \mathbf{F}_{\text{clk}} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{Q}_{\text{clk},r} = c^2 \begin{bmatrix} S_{\tilde{w}_{\delta t_r}} T + S_{\tilde{w}_{\dot{\delta}t_r}} \frac{T^3}{3} & S_{\tilde{w}_{\delta t_r}} \frac{T^2}{2} \\ S_{\tilde{w}_{\dot{\delta}t_r}} \frac{T^2}{2} & S_{\tilde{w}_{\delta t_r}} T \end{bmatrix}$$

$$\mathbf{Q}_{\text{pv}} = \begin{bmatrix} \tilde{q}_x \frac{T^3}{3} & 0 & \tilde{q}_x \frac{T^2}{2} & 0 \\ 0 & \tilde{q}_y \frac{T^3}{3} & 0 & \tilde{q}_y \frac{T^2}{2} \\ \tilde{q}_x \frac{T^2}{2} & 0 & \tilde{q}_x T & 0 \\ 0 & \tilde{q}_y \frac{T^2}{2} & 0 & \tilde{q}_y T \end{bmatrix}$$

where  $\tilde{q}_x$  and  $\tilde{q}_y$  are the power spectral densities of the continuous-time  $x$  and  $y$  acceleration noise, respectively. The spectra  $S_{\tilde{w}_{\delta t_r}}$  and  $S_{\tilde{w}_{\dot{\delta}t_r}}$  are modeled similarly to the BTS spectra, but with receiver-specific  $h_{0,r}$  and  $h_{-2,r}$ .

### C. Cellular Carrier Phase Measurement Model

A specialized receiver (e.g., [3], [7], [9], [10]) could produce a carrier phase observable to the  $n$ th BTS given by

$$\phi_n(t) = \phi_n(t_0) + \int_{t_0}^t f_{D,n}(\tau) d\tau, \quad n = 1, \dots, N \quad (1)$$

where  $\phi_n(t_0)$  is the carrier phase at an initial time  $t_0$  and  $f_{D,n}(\tau)$  is the Doppler frequency at time  $\tau$ . The carrier phase observable in (1) could be parameterized in terms of the receiver and BTS states to yield the discrete-time measurement model given by

$$z_n(k) \triangleq \lambda \phi_n(t_0 + kT) = \|\mathbf{r}_r(k) - \mathbf{r}_{s_n}\|_2 + c [\delta t_r(k) - \delta t_{s_n}(k)] + \lambda N_n + v_n(k) \quad (2)$$

where  $\lambda$  is the wavelength of the carrier signal,  $N_n$  represents the carrier phase ambiguity corresponding to the  $n$ th BTS (namely, the initial phase difference between the receiver and the  $n$ th BTS), and  $v_n(k)$  is the measurement noise, which is modeled as a discrete-time zero-mean white Gaussian sequence with variance  $\sigma_n^2(k)$ . Note that the measurements are uncorrelated in time because it is assumed that 1) the sampling time is larger than the channel coherence time and 2) the channels between the UAV and the cellular BTSs are multipath free. The second practical assumption is made, since a strong line-of-sight component is usually observed in the received signal [12].

### III. NAVIGATION WITH CELLULAR SOP CARRIER PHASE MEASUREMENTS

This section formulates an EKF-based framework for standalone navigation with carrier phase measurements from asynchronous BTSs.

#### A. Motivation

In GNSS, the satellites' positions and clock biases are known to the receiver. Subsequently, with  $N \geq 4$  satellites, one can use a static estimator (e.g., least squares) to solve for the receiver's 3-D position and its clock bias without the need of a dynamic estimator. A similar approach can be taken in a generic radionavigation systems, where the receiver is making pseudorange-type measurements and the navigation source locations (i.e., transmitters) and their clock biases are known to the estimator. This article considers the case of opportunistic navigation with cellular signals, where BTSs may be asynchronous and have unknown clock biases. As a result, one has to account for the receiver and BTS clock biases, leading to an underdetermined system. To address this, a dynamic estimator must be employed to obtain a navigation solution. In this article, an EKF is used as a navigation filter. The EKF relies on the dynamics model of the receiver motion and the clock errors. With minimal knowledge of the state dynamics, one can use kinematic models to propagate the state estimate between measurement updates [30]. The choice of the kinematic model order depends on the application. In the case of a maneuvering target, i.e., when there is some control input on a derivative of the position coordinates, this control input can be made available to the EKF, since it is generated by the vehicle's on-board controller. As a result, the EKF implementation will not change except for incorporating the control input in the state time-update equations. The next subsections describe a detailed implementation of an EKF that estimates the state of the system defined in (3) and (4).

#### B. Modified Clock Error States

Estimating the terms  $c\delta t_r(k)$ ,  $c\delta t_{s_n}(k)$ , and  $\lambda N_n$  in (2) individually is unnecessary; hence, they will be lumped into one bias term defined as

$$c\delta t_n(k) \triangleq c \left[ \delta t_r(k) - \delta t_{s_n}(k) + \frac{\lambda}{c} N_n \right]$$

with an associated drift state  $c\delta t_n(k)$  given by

$$c\dot{\delta t}_n(k) \triangleq c [\dot{\delta t}_r(k) - \dot{\delta t}_{s_n}(k)].$$

One may subsequently conclude that the dynamics of  $\mathbf{x}_{\text{clk},n}(k) \triangleq [c\delta t_n(k), c\dot{\delta t}_n(k)]^T$  is given by

$$\mathbf{x}_{\text{clk},n}(k+1) = \mathbf{F}_{\text{clk}} \mathbf{x}_{\text{clk},n}(k) + \mathbf{w}_{\text{clk},n}(k), \quad n = 1, \dots, N$$

where  $\mathbf{w}_{\text{clk},n}(k)$  is a discrete-time zero-mean white noise sequence with covariance  $\mathbf{Q}_{\text{clk},n} = \mathbf{Q}_{\text{clk},r} + \mathbf{Q}_{\text{clk},s_n}$ . Note that now  $\mathbf{w}_{\text{clk},n}(k)$  and  $\mathbf{w}_{\text{clk},m}(k)$  are correlated, with

$$\mathbb{E} [\mathbf{w}_{\text{clk},n}(k) \mathbf{w}_{\text{clk},m}^T(k)] = \begin{cases} \mathbf{Q}_{\text{clk},n}, & \text{if } n = m \\ \mathbf{Q}_{\text{clk},r}, & \text{otherwise.} \end{cases}$$

#### C. EKF Model

The EKF estimates the UAV-mounted receiver's position and velocity and the modified clock error states for all BTSs, namely

$$\mathbf{x}(k) \triangleq [\mathbf{r}_r^T(k), c\delta t_1(k), \dots, c\delta t_N(k), \dot{\mathbf{r}}_r^T(k), c\dot{\delta t}_1(k), \dots, c\dot{\delta t}_N(k)]^T.$$

Note that  $\mathbf{x}(k)$  may be expressed as  $\mathbf{x}(k) = \mathbf{\Pi} \mathbf{x}'(k)$ , where  $\mathbf{x}'(k) \triangleq [\mathbf{r}_r^T(k), \dot{\mathbf{r}}_r^T(k), \mathbf{x}_{\text{clk},1}^T(k), \dots, \mathbf{x}_{\text{clk},N}^T(k)]^T$  and  $\mathbf{\Pi}$  is some permutation matrix that could be readily calculated. The EKF considers the system with the following dynamics and measurement model:

$$\mathbf{x}(k+1) = \mathbf{F} \mathbf{x}(k) + \mathbf{w}(k) \quad (3)$$

$$\mathbf{z}(k) = \mathbf{h}[\mathbf{x}(k)] + \mathbf{v}(k) \quad (4)$$

with  $\mathbf{h}[\mathbf{x}(k)] \triangleq [h_1[\mathbf{x}(k)], \dots, h_N[\mathbf{x}(k)]]^T$ ,  $h_n[\mathbf{x}(k)] \triangleq \|\mathbf{r}_r(k) - \mathbf{r}_{s_n}\| + c\delta t_n(k)$ ,  $\mathbf{z}(k) \triangleq [z_1(k), \dots, z_N(k)]^T$ ,  $\mathbf{w}(k)$  is a discrete-time zero-mean white sequence with covariance  $\mathbf{Q} \triangleq \mathbf{\Pi} \mathbf{Q}' \mathbf{\Pi}^T$ , where  $\mathbf{Q}' \triangleq \text{diag}[\mathbf{Q}_{pv}, \mathbf{Q}_{\text{clk}}]$ ,

$$\mathbf{Q}_{\text{clk}} \triangleq \begin{bmatrix} \mathbf{Q}_{\text{clk},1} & \mathbf{Q}_{\text{clk},r} & \dots & \mathbf{Q}_{\text{clk},r} \\ \mathbf{Q}_{\text{clk},r} & \mathbf{Q}_{\text{clk},2} & \dots & \mathbf{Q}_{\text{clk},r} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Q}_{\text{clk},r} & \mathbf{Q}_{\text{clk},r} & \dots & \mathbf{Q}_{\text{clk},N} \end{bmatrix},$$

$$\mathbf{F} \triangleq \begin{bmatrix} \mathbf{I}_{p \times p} & T \mathbf{I}_{p \times p} \\ \mathbf{0}_{p \times p} & \mathbf{I}_{p \times p} \end{bmatrix}$$

with  $p = N + 2$ , and  $\mathbf{v}(k) \triangleq [v_1(k), \dots, v_N(k)]^T$  is a discrete-time zero-mean white Gaussian sequence with covariance  $\mathbf{R}(k) \triangleq \text{diag}[\sigma_1^2(k), \dots, \sigma_N^2(k)]$ . Section VI discusses how the process and measurement noise covariance matrices  $\mathbf{Q}$  and  $\mathbf{R}(k)$ , respectively, are selected in a practical environment.

The EKF is producing an estimate  $\hat{\mathbf{x}}(k|j) = \mathbb{E}[\mathbf{x}(k)|z(1), \dots, z(j)]$ ,  $j \leq k$ , with an associated estimation error covariance  $\mathbf{P}(k|j) = \mathbb{E}[\tilde{\mathbf{x}}(k|j) \tilde{\mathbf{x}}^T(k|j)]$ , where  $\tilde{\mathbf{x}}(k|j) \triangleq \mathbf{x}(k) - \hat{\mathbf{x}}(k|j)$  is the estimation error. The current state estimate  $\hat{\mathbf{x}}(k|k)$  and its associated estimation error covariance  $\mathbf{P}(k|k)$  are obtained using the standard EKF equations. The measurement Jacobian  $\mathbf{H}(k)$  used in the EKF estimation error covariance update is given by

$$\mathbf{H}(k) = [\mathbf{G}(k) \quad \mathbf{I}_{N \times N} \quad \mathbf{0}_{(N+2) \times (N+2)}] \quad (5)$$

$$\mathbf{G}(k) \triangleq \begin{bmatrix} \frac{\mathbf{r}_r(k) - \mathbf{r}_{s_1}}{\|\mathbf{r}_r(k) - \mathbf{r}_{s_1}\|} & \dots & \frac{\mathbf{r}_r(k) - \mathbf{r}_{s_N}}{\|\mathbf{r}_r(k) - \mathbf{r}_{s_N}\|} \end{bmatrix}^T \quad (6)$$

where  $\mathbf{G}(k)$  is evaluated at  $\hat{\mathbf{x}}(k|j)$ .

#### D. EKF Initialization

It is assumed that the UAV has access to GNSS signals at  $k = 0$  and  $k = 1$ , from which it could estimate its position. These position estimates, denoted by  $\mathbf{z}_r(0)$  and  $\mathbf{z}_r(1)$ , can be modeled as

$$\mathbf{z}_r(j) = \mathbf{r}_r(j) + \mathbf{v}_r(j), \quad j = 0, 1 \quad (7)$$

where  $\mathbf{r}_r(j)$  is the UAV's true position and  $\mathbf{v}_r(j)$  is a random vector that captures the estimation uncertainty, which is modeled as a zero-mean white Gaussian random sequence with covariance  $\Sigma_{\mathbf{r}_r}(j)$ . Moreover, during the same time GNSS signals are available, the receiver makes two consecutive carrier phase measurements to the  $N$  available BTSs prior to initializing the EKF, denoted by  $\mathbf{z}(0)$  and  $\mathbf{z}(1)$ . The maximum likelihood (ML) estimate of  $\mathbf{x}(1)$  from  $\mathbf{z}_{\text{ini}} \triangleq [\mathbf{z}_{r_r}^T(1), \mathbf{z}_{r_r}^T(0), \mathbf{z}^T(1), \mathbf{z}^T(0)]^T$  is obtained according to Appendix A, yielding  $\hat{\mathbf{x}}_{\text{ML,ini}}$  and its associated estimation error covariance  $\mathbf{P}_{\text{ML,ini}}$ . Finally, the EKF is initialized with

$$\hat{\mathbf{x}}(1|1) \equiv \hat{\mathbf{x}}_{\text{ML,ini}}, \quad \mathbf{P}(1|1) \equiv \mathbf{P}_{\text{ML,ini}}$$

and is run for  $k \geq 1$ .

REMARK 1 Let  $v_{r_1}$  and  $\hat{v}_{r_1}$  denote the true initial speed and its estimate, respectively, and let  $\mathbf{u}_{r_1}$  and  $\hat{\mathbf{u}}_{r_1}$  denote the true initial velocity direction unit vector and its estimate, respectively. Let  $\sigma_{v_{r_1}}^2$  and  $\mathbf{P}_{\mathbf{u}_{r_1}}$  denote the initial variance and covariance of the initial estimation errors  $\tilde{v}_{r_1} \triangleq v_{r_1} - \hat{v}_{r_1}$  and  $\tilde{\mathbf{u}}_{r_1} \triangleq \mathbf{u}_{r_1} - \hat{\mathbf{u}}_{r_1}$ , respectively. It is shown in Appendix B that

$$\sigma_{v_{r_1}}^2 \leq \frac{\lambda_{\max, \dot{r}}}{T^2}, \quad \sigma_{\mathbf{u}_{r_1}}^2 \triangleq \|\mathbf{P}_{\mathbf{u}_{r_1}}\| \leq \frac{\lambda_{\max, \dot{r}}}{\|\mathbf{r}_r(1) - \mathbf{r}_r(0)\|^2} \quad (8)$$

where  $\lambda_{\max, \dot{r}}$  is a positive real number that only depends on  $\tilde{\Sigma}_{\mathbf{r}_r} \triangleq \Sigma_{\mathbf{r}_r}(0) + \Sigma_{\mathbf{r}_r}(1)$ . It can be seen from (8) that increasing  $T$  reduces the uncertainty in the initial speed estimate. Increasing  $\|\mathbf{r}_r(1) - \mathbf{r}_r(0)\|$  improves the estimate of the initial velocity direction unit vector. Given  $\Sigma_{\mathbf{r}_r}(0)$  and  $\Sigma_{\mathbf{r}_r}(1)$ , (8) may be used to choose an initial sampling time  $T$  and/or distance  $\|\mathbf{r}_r(1) - \mathbf{r}_r(0)\|$  that guarantee that  $\sigma_{v_{r_1}}^2$  and  $\sigma_{\mathbf{u}_{r_1}}^2$  will be below a specified threshold. Regardless of the choice of  $T$  and  $\|\mathbf{r}_r(1) - \mathbf{r}_r(0)\|$ , the proposed initialization scheme will always yield an initial state estimate that is consistent with its initial estimation error covariance, which is important when initializing the EKF.

#### IV. OBSERVABILITY AND EKF ESTIMATION ERROR BOUNDS ANALYSES

This section shows that the system defined in (3) and (4) is observable for  $N \geq 2$ . Moreover, it shows that the EKF estimation error is exponentially bounded in the mean square sense and bounded with probability 1. In the following, the following assumptions are made.

- A1) The BTSs are not colocated nor are all collinear.
- A2) The UAV is not stationary nor is moving along a trajectory that is collinear with the vector connecting its receiver with any of the BTSs.
- A3) The UAV is at a minimum distance  $d$  from each BTS at all time, i.e.,  $\|\mathbf{r}(k) - \mathbf{r}_{s_n}\| \geq d, \forall k > 0$  and  $\forall n = 1, \dots, N$ .

This article aims at qualitatively studying the behavior of the EKF estimation error and estimation error covariance. To this end, generic theorems and lemmas on

EKF stochastic stability are applied to the opportunistic navigation system at hand, which is implemented in a real-world environment. The relevant theorems and lemmas are stated in Appendix C, which gives theoretical background on observability and boundedness of the EKF error state.

##### A. Observability Analysis

The observability of an environment comprising multiple receivers making pseudorange measurements on multiple BTSs, assuming different *a priori* knowledge scenarios, was analyzed in [31]. The observability analysis utilized the  $l$ -step observability matrix of the linearized system and considered the observability of the *individual* clock biases and drifts  $c\delta t_r(k), c\dot{\delta}t_r(k), \{c\delta t_{s_n}(k)\}_{n=1}^N$ , and  $\{c\dot{\delta}t_{s_n}(k)\}_{n=1}^N$ .

In contrast, the system in (3) and (4) considers a single receiver making carrier phase measurements on multiple BTSs, where the individual clock biases and carrier phase ambiguities are lumped into a *single* bias term  $\{c\delta t_n(k)\}_{n=1}^N$  and the drifts are also lumped into a *single* drift term  $\{c\dot{\delta}t_n(k)\}_{n=1}^N$ .

The observability results for the system defined in (3) and (4) are captured in the following theorem.

**THEOREM IV.1** Under Assumptions A1 and A2, the system defined in (3) and (4) is completely  $l$ -step observable for  $l \geq 4$  and  $N \geq 2$ .

**PROOF** The linearization of the deterministic part of the system (3), (4) into the form (47), (48) yields

$$\mathbf{F}(k) \equiv \begin{bmatrix} \mathbf{I}_{(N+2) \times (N+2)} & T\mathbf{I}_{(N+2) \times (N+2)} \\ \mathbf{0}_{(N+2) \times (N+2)} & \mathbf{I}_{(N+2) \times (N+2)} \end{bmatrix}, \quad \mathbf{\Gamma}(k) \equiv \mathbf{0}$$

$$\mathbf{H}(k) \equiv [\mathbf{H}_\xi(k) \quad \mathbf{0}_{N \times (N+2)}], \quad \mathbf{H}_\xi(k) \triangleq [\mathbf{G}(k) \quad \mathbf{I}_{N \times N}].$$

In the following, it will be proven by construction that the  $l$ -step observability matrix  $\mathcal{O}(k, k+l)$  of the linearized system is full rank, i.e.,

$$\sum_{i=1}^{2L} \gamma_i \mathcal{O}(k, k+l) \mathbf{e}_{2L,i} = \mathbf{0} \quad (9)$$

is satisfied *if and only if*  $\gamma_i = 0, \forall i = 1, \dots, 2L$ , where  $L = N + 2$  and  $\mathbf{e}_{L,i} \in \mathbb{R}^L$  is the standard basis vector consisting of a one at the  $i$ th element and zeros elsewhere. Note that since  $\mathcal{O}(k, k+l) \in \mathbb{R}^{L \times 2(N+2)}$ , then  $l \geq 4$  always satisfies  $l \cdot N \geq 2(N+2)$  for  $N \geq 2$ . Let  $l = 4$ . Subsequently,  $\mathcal{O}(k, k+4)$  may be expressed as

$$\mathcal{O}(k, k+4) = \begin{bmatrix} \mathcal{O}_{11} & \mathcal{O}_{12} \\ \mathcal{O}_{21} & \mathcal{O}_{22} \end{bmatrix}$$

$$\mathcal{O}_{11} \triangleq \begin{bmatrix} \mathbf{H}_\xi(k) \\ \mathbf{H}_\xi(k+1) \end{bmatrix}, \quad \mathcal{O}_{21} \triangleq \begin{bmatrix} \mathbf{H}_\xi(k+2) \\ \mathbf{H}_\xi(k+3) \end{bmatrix}$$

$$\mathcal{O}_{12} \triangleq \begin{bmatrix} \mathbf{0} \\ T\mathbf{H}_\xi(k+1) \end{bmatrix}, \quad \mathcal{O}_{22} \triangleq \begin{bmatrix} 2T\mathbf{H}_\xi(k+2) \\ 3T\mathbf{H}_\xi(k+3) \end{bmatrix}.$$

The matrix  $\mathcal{O}_{11}$  may also be expressed as

$$\mathcal{O}_{11} = \begin{bmatrix} \mathbf{G}(k) & \mathbf{I}_{N \times N} \\ \mathbf{G}(k+1) & \mathbf{I}_{N \times N} \end{bmatrix}. \quad (10)$$

Note that  $\mathcal{O}_{11} \in \mathbb{R}^{2N \times (N+2)}$ . Moreover, the inequality  $2N \geq N + 2$  holds, for  $N \geq 2$ . Therefore, we have

$$\text{rank}[\mathcal{O}_{11}] \leq N + 2. \quad (11)$$

From (10), it can be seen that  $\text{rank}[\mathcal{O}_{11}] \geq N$ . Moreover, for  $N \geq 2$ , and if A1 and A2 hold, then the  $(N + 1)$ th and  $(N + 2)$ th rows of  $\mathcal{O}_{11}$  will be linearly independent from the first  $N$  rows and from each other, yielding

$$\text{rank}[\mathcal{O}_{11}] \geq N + 2. \quad (12)$$

Combining (11) and (12), it can be deduced that  $\text{rank}[\mathcal{O}_{11}] = N + 2$ . Similarly, it can be shown that  $\text{rank}[\mathcal{O}_{21}] = N + 2$ . Subsequently, we have that

$$\sum_{i=1}^L \alpha_i \mathcal{O}_{11} \mathbf{e}_{L,i} = \mathbf{0}, \quad \sum_{i=1}^L \beta_i \mathcal{O}_{21} \mathbf{e}_{L,i} = \mathbf{0}$$

are satisfied *if and only if*  $\alpha_i = \beta_i = 0, \forall i = 1, \dots, L$ . Therefore, the equality

$$\sum_{i=1}^L \pi_i \mathbf{H}_\xi(k+j) \mathbf{e}_{L,i} = \mathbf{0} \quad \forall j = 0, \dots, 3 \quad (13)$$

is satisfied *if and only if*  $\pi_i = 0, \forall i = 1, \dots, L$ .

The left-hand side of (9) can be expressed as

$$\sum_{i=1}^{2L} \gamma_i \mathcal{O}(k, k+4) \mathbf{e}_{2L,i} = [\boldsymbol{\rho}_0^\top, \dots, \boldsymbol{\rho}_3^\top]^\top \quad (14)$$

$$\boldsymbol{\rho}_j \triangleq \sum_{i=1}^L (\gamma_i + jT \gamma_{L+i}) \mathbf{H}_\xi(k+j) \mathbf{e}_{L,i} \quad (15)$$

where  $j = 0, \dots, 3$ . It can be seen from (13) that  $\boldsymbol{\rho}_j = \mathbf{0}$  for all  $j = 0, \dots, 3$  *if and only if*

$$\gamma_i + jT \gamma_{L+i} = 0 \quad \forall i = 1, \dots, L; \quad \forall j = 0, \dots, 3. \quad (16)$$

Since (16) holds for all  $j = 0, \dots, 3$ , then evaluating (16) at  $j = 0$  yields

$$\gamma_i = 0 \quad \forall i = 1, \dots, L. \quad (17)$$

Combining (16) and (17) for  $j > 0$  yields

$$\gamma_{L+i} = 0 \quad \forall i = 1, \dots, L. \quad (18)$$

Equations (16)–(18) imply (9); therefore,  $\mathcal{O}(k, k+4)$  is full rank. Using Theorem VII.1 in Appendix C, the system is observable. ■

**REMARK 2** Note that the  $l$ -step observability matrix  $\mathcal{O}(k, k+l)$  is an  $l \cdot N \times 2(N+2)$  matrix; hence,  $\text{rank}[\mathcal{O}(k, k+l)] \leq \min\{l \cdot N, 2(N+2)\}$ . Subsequently, one necessary condition for the observability matrix to be full rank is that  $l \cdot N \geq 2(N+2)$ , i.e., the UAV makes carrier phase measurements at  $l$  epochs to the  $N$  cellular BTSs. For  $N \geq 2$ , this condition is satisfied for  $l \geq 4$ . For  $N \geq 4$ , this condition is satisfied for  $l \geq 3$ .

**REMARK 3** The result of Theorem IV.1 is only valid locally and in a deterministic sense, i.e., with no process or measurement noise. However, this result can be extended to the stochastic system (52), (53). Let the measurement Jacobian  $\mathbf{G}(k)$  with respect to the position states [cf. (6)] be reparameterized in terms of the bearing angles  $\{\theta_n(k)\}_{n=1}^N$  between each BTS and the UAV according to

$$\mathbf{G}(k) = \begin{bmatrix} \cos[\theta_1(k)] & \dots & \cos[\theta_N(k)] \\ \sin[\theta_1(k)] & \dots & \sin[\theta_N(k)] \end{bmatrix}^\top.$$

The presence of process noise will yield new bearing angle trajectories  $\theta'_n(k) = \theta_n(k) + \delta\theta_n(k)$ , where  $\delta\theta_n(k)$  is the bearing angle error due to process noise. With assumptions A.1 and A.2, the new bearing angles will not change the structure nor the rank of  $\mathbf{H}(k)$  [cf. (5)], which will remain a combination of cosine and sine functions and other constants. This, in turn, will satisfy the observability condition of the system with process noise. More details can be found in [32, Lemma 4.1] and [33, Corollary 5.2].

**REMARK 4** The velocity random walk model considered in this article is simple yet informative enough to capture the UAV dynamics between the EKF measurement updates. In particular, the EKF will use this model to perform the time-update step, and for sufficiently small  $T$ , the UAV's dynamics model would not deviate drastically from this model. Nevertheless, this model may not necessarily cover the variety of flight modes that are achievable using typical rotary wing UAVs. To address this, one may employ a multiple model approach to estimate the UAV's state, with each model matched to a different flight mode [30], [34]–[36]. The observability analysis presented in this article can be performed for each of the dynamics model assumed by the multiple model filter, such as acceleration random walk, constant turn rate, or even a stationary UAV. Theorem IV.1 can be extended to study the observability of higher order UAV dynamics models (e.g., acceleration random walk, jerk random walk, etc. [29]) and higher order clock error dynamics models (e.g., three-state clock error dynamics comprising time, frequency, and frequency aging [37], [38]). It can be readily shown that the system becomes unobservable when the UAV stops.

## B. Lower Bound on the EKF Estimation Error Covariance

The optimal geometric configuration of sensors (or navigation sources) around an emitter (or receiver) has been well studied in the literature [39], [40]. It was found that in the presence of independent and identically distributed measurement noise, the trace of the estimation error covariance in a nonlinear least-squares estimator is minimized when the end points of the unit line of sight vectors pointing from the receiver to each navigation source form a regular polygon around the receiver, i.e.,  $\theta_n = \frac{2\pi(n-1)}{N}, n = 1, \dots, N \geq 3$  [41]. The aforementioned configuration will be referred to as the optimal configuration.

Although the system discussed in Section III-C is nonlinear, one may devise a scenario for  $N \geq 3$  that will define

a lower bound (LB) on the estimation error covariance in the EKF. To this end, it is assumed that the optimal BTS configuration around the receiver is maintained at all time, implying that Assumption A1 is satisfied. Assumption A2 implies that the measurement Jacobian cannot be the same at all time. In order to satisfy A1 and A2 simultaneously, it is assumed that the optimal configuration is maintained and that the BTSs rotate around the receiver on the unit circle by  $2\pi/N$  at each time step. Therefore, the optimal bearing angles at any given time step  $k$  will be given by

$$\theta_n^*(k) = \frac{2\pi \cdot \text{mod}(n-1+k, N)}{N}, \quad n=1, \dots, N, \quad k=1, \dots$$

where  $\text{mod}(\cdot, \cdot)$  is the modulo operator. Note that this parameterization is independent of the state. Therefore, the Riccati equation may be iterated offline with the optimal configuration and measurement noise covariance  $\mathbf{R}^*(k) \equiv \lambda_{\min}[\mathbf{R}(k)]\mathbf{I}$  to produce an LB on the estimation error covariance for the EKF, denoted  $\mathbf{P}_{\min}(k+1|k)$ , from which a real number  $\underline{p} > 0$  such that  $\underline{p}\mathbf{I} \leq \mathbf{P}_{\min}(k+1|k)$  can be deduced. Note that  $\lambda_{\min}[\mathbf{A}]$  indicates the smallest eigenvalue of matrix  $\mathbf{A}$ . It is also important to note that while this scenario could never be physically realized, it is only used to define an LB on the estimation error covariance.

**REMARK 5** The intuition behind obtaining this LB is explained next. Assume two configurations for  $N$  BTSs: (i) the optimal one and (ii) any other arbitrary configuration. Given the same prior for both configurations, i.e.,  $\mathbf{P}_i(k|k) = \mathbf{P}_{ii}(k|k)$ , then  $\mathbf{P}_i(k+1|k) = \mathbf{P}_{ii}(k+1|k)$ , since the dynamics are linear time-invariant. The covariance measurement updates can be expressed in the information form as

$$\begin{aligned} \mathbf{P}_i^{-1}(k+1|k+1) &= \mathbf{P}_i^{-1}(k+1|k) \\ &+ \frac{1}{\sigma^2} \mathbf{H}_i^T(k+1) \mathbf{H}_i(k+1) \end{aligned} \quad (19)$$

$$\begin{aligned} \mathbf{P}_{ii}^{-1}(k+1|k+1) &= \mathbf{P}_{ii}^{-1}(k+1|k) \\ &+ \frac{1}{\sigma^2} \mathbf{H}_{ii}^T(k+1) \mathbf{H}_{ii}(k+1) \end{aligned} \quad (20)$$

Since (i) is the optimal configuration, then

$$\mathbf{H}_i^T(k+1) \mathbf{H}_i(k+1) \geq \mathbf{H}_{ii}^T(k+1) \mathbf{H}_{ii}(k+1). \quad (21)$$

From (19)–(21), it can be seen that  $\mathbf{P}_i(k+1|k+1) \leq \mathbf{P}_{ii}(k+1|k+1)$ . Repeating this recursion yields  $\mathbf{P}_i(k+j|k+j) \leq \mathbf{P}_{ii}(k+j|k+j)$  for all  $j \geq 1$ .

### C. EKF Estimation Error Bounds Analysis

Consider the following two notions of estimation error boundedness defined in [32].

**DEFINITION IV.1** The stochastic sequence  $\tilde{\mathbf{x}}(k|k)$  is said to be exponentially bounded in mean square, if there are real numbers  $\eta, \nu > 0$  and  $0 < \vartheta < 1$  such that

$$\mathbb{E} [\|\tilde{\mathbf{x}}(k|k)\|^2] \leq \eta \|\tilde{\mathbf{x}}(1|1)\|^2 \vartheta^k + \nu \quad (22)$$

holds for every  $k > 0$ .

**DEFINITION IV.2** The stochastic sequence  $\tilde{\mathbf{x}}(k|k)$  is said to be exponentially bounded with probability 1, if

$$\sup_{k>0} \|\tilde{\mathbf{x}}(k|k)\| < \infty \quad (23)$$

holds with probability 1.

Next, it is shown that the EKF estimation error for the system at hand is bounded according to Definitions IV.1 and IV.2. From the system defined in (3) and (4), it can be seen that  $\mathbf{F}(k) = \mathbf{F}$  is nonsingular and

$$\|\mathbf{F}\| = 1 \quad (24)$$

for all  $k \geq 0$ . Moreover, from the definition of  $\mathbf{Q}$  and  $\mathbf{R}(k)$  in Section III-C, it can be seen that  $\mathbf{Q}(k) = \mathbf{Q} > \mathbf{0}$  and  $\mathbf{R}(k) > \mathbf{0}$ ; hence, there exist real numbers  $\underline{q}, \underline{r} > 0$  such that

$$\mathbf{Q} \geq \underline{q}\mathbf{I}, \quad \mathbf{R}(k) \geq \underline{r}\mathbf{I} \quad (25)$$

for all  $k > 0$ . It was established in Theorem IV.1 that the system is observable; hence, using Lemma C.1 in Appendix C, there exist real numbers  $\underline{p}, \bar{p} > 0$  such that

$$\underline{p}\mathbf{I} \leq \mathbf{P}(k+1|k) \leq \bar{p}\mathbf{I} \quad \forall k > 0. \quad (26)$$

An approach for obtaining  $\underline{p}$  is given in Section IV-B. Since the dynamics of the system in (3) are linear, then

$$\|\varphi(k)\| = 0 \quad \forall k > 0. \quad (27)$$

The following two lemmas establish the rest of the conditions for Theorem VII.2 in Appendix C to hold.

**LEMMA IV.1** The 2-norm of the measurement Jacobian defined in (5) is bounded by

$$\|\mathbf{H}(k)\| \leq \sqrt{N+1} \quad \forall k > 0. \quad (28)$$

**PROOF** Equation (28) follows from showing that

$$\mathbf{H}^T(k) \mathbf{H}(k) \leq (N+1)\mathbf{I}. \quad (29)$$

The matrix  $\mathbf{\Delta} \triangleq (N+1)\mathbf{I} - \mathbf{H}^T(k) \mathbf{H}(k)$  is expressed as

$$\begin{aligned} \mathbf{\Delta} &= \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & (N+1)\mathbf{I} \end{bmatrix} \\ \mathbf{M} &\triangleq \begin{bmatrix} (N+1)\mathbf{I} - \mathbf{G}^T(k) \mathbf{G}(k) & -\mathbf{G}^T(k) \\ -\mathbf{G}(k) & N\mathbf{I} \end{bmatrix} \end{aligned}$$

which implies that (29) is satisfied when  $\mathbf{M} \geq \mathbf{0}$ . Since  $N\mathbf{I} > \mathbf{0}$ , then  $\mathbf{M}$  is positive semidefinite if the Schur complement of its bottom-right block given by

$$\mathbf{M}_{\text{Schur}} \triangleq (N+1)\mathbf{I} - \mathbf{G}^T(k) \mathbf{G}(k) - \frac{1}{N} \mathbf{G}^T(k) \mathbf{G}(k)$$

is positive semidefinite. For any matrix  $\mathbf{A}$ , the following holds:

$$\mathbf{A}^T \mathbf{A} \leq \text{trace}[\mathbf{A}^T \mathbf{A}] \mathbf{I}.$$

It can be readily shown that  $\text{trace}[\mathbf{G}^T(k) \mathbf{G}(k)] = N$  for all  $k \geq 0$ . Subsequently, we have

$$\mathbf{M}_{\text{Schur}} \geq (N+1)\mathbf{I} - N\mathbf{I} - \mathbf{I} = \mathbf{0} \quad \Rightarrow \quad \mathbf{M} \geq \mathbf{0}$$

from which (29) and consequently (28) follow.  $\blacksquare$

LEMMA IV.2 Consider the system defined in (3) and (4). If A3 holds, then

$$\max_{1 \leq n \leq N} \sup_{\mathbf{x}(k)} \|\text{Hess } h_n[\mathbf{x}(k)]\| \leq \frac{1}{d} \quad (30)$$

where Hess denotes the Hessian operator.

PROOF It can be readily shown that

$$\text{Hess } h_n[\mathbf{x}(k)] = \frac{1}{\|\mathbf{r}_r(k) - \mathbf{r}_{s_n}\|} \text{diag}[\mathbf{U}, \mathbf{0}_{(2N+2) \times (2N+2)}]$$

where  $\mathbf{U} \triangleq \mathbf{I}_{2 \times 2} - \mathbf{v}\mathbf{v}^\top$  and  $\mathbf{v} \triangleq \frac{\mathbf{r}_r(k) - \mathbf{r}_{s_n}}{\|\mathbf{r}_r(k) - \mathbf{r}_{s_n}\|}$ . It can be seen that the matrix  $\mathbf{U}$  is an annihilator matrix, and therefore, its eigenvalues consist of ones and zeros. Subsequently, we have

$$\|\text{Hess } h_n[\mathbf{x}(k)]\| = \frac{1}{\|\mathbf{r}_r(k) - \mathbf{r}_{s_n}\|}.$$

Since A3 holds, i.e.,  $\|\mathbf{r}_r(k) - \mathbf{r}_{s_n}\| \geq d$ , then  $\|\text{Hess } h_n[\mathbf{x}(k)]\| \leq \frac{1}{d}$ , which in turn implies (30). ■

Using Taylor's theorem and Lemma IV.2, it can be deduced that

$$\|\chi(k)\| \leq \kappa_\chi \|\tilde{\mathbf{x}}(k|k)\|^2 \quad (31)$$

where  $\kappa_\chi = \frac{1}{d}$  [32]. Now, the main result for the EKF error bounds is stated.

THEOREM IV.2 Consider the system defined in (3) and (4) whose state is being estimated using an EKF as described in Section III-C. If A1–A3 hold, then the EKF error  $\tilde{\mathbf{x}}(k|k)$  is exponentially bounded in the mean square and bounded with probability 1 as per Definitions IV.1 and IV.2, respectively, assuming

$$\|\tilde{\mathbf{x}}(1|1)\| \leq \epsilon, \quad \mathbf{R}(k) \leq \delta \mathbf{I}, \quad \mathbf{Q} \leq \delta \mathbf{I}$$

for some  $\epsilon, \delta > 0$ .

PROOF Combining (24)–(28), (31), and the fact that  $\mathbf{F}$  is nonsingular, one can see that all the conditions of Theorem C.2 in Appendix C are satisfied, from which one concludes that  $\tilde{\mathbf{x}}(k|k)$  is exponentially bounded and bounded with probability 1. ■

The significance of the results presented in Theorems IV.1 and IV.2 are in the fact that it is possible for a UAV to reliably navigate for long periods of time exclusively with cellular signals transmitted by asynchronous BTSs.

REMARK 6 While the observability and EKF stability analysis establishes bounds on the estimation error covariance, it does not specify how to obtain such bounds. However, it is worth noting that the observability analysis can be extended for higher order models or for models with known control input. It can also be shown that the system will be  $l$ -step observable for such models. Subsequently, the EKF output covariance for these dynamics models will be bounded as well. As a result, Theorem IV.2 can be readily extended to show that an EKF using kinematic models with

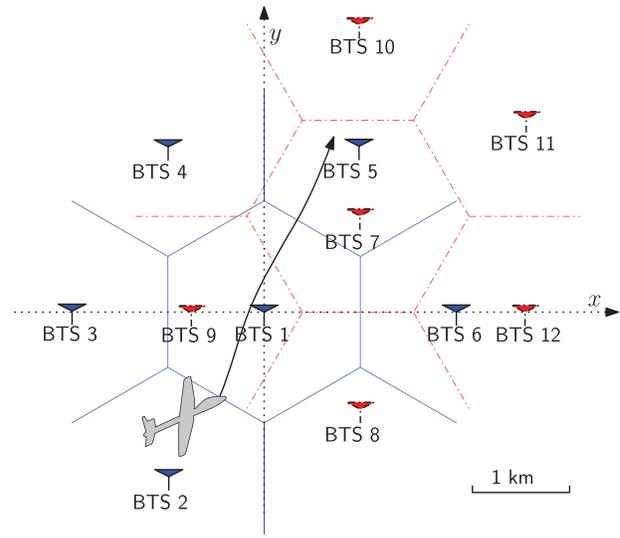


Fig. 1. Simulation environment layout. The blue and red colors represent cells and BTSs from two different cellular providers. The UAV's trajectory is shown in black.

or without known control inputs will have bounded errors as per Definitions IV.1 and IV.2.

## V. SIMULATION RESULTS

This section presents simulations to analyze the performance of the proposed EKF framework by varying: 1) the number of available BTSs; 2) the initial speed of the UAV; and 3) the quality of the oscillator on-board the UAV. The simulation setup and settings are discussed first; then, the results are provided.

### A. Simulation Setup

The simulated environment consisted of 12 BTSs from two cellular providers and is illustrated in Fig. 1. It can be seen that due to the cellular structure, the geometry between the UAV and BTSs is favorable. The cell size was picked to be 2 km, which is a typical value in semiurban environments. MC simulations were ran for different values of the number of available BTSs  $N$ , the initial speed of the UAV  $v_{r_1}$ , and the UAV-mounted receiver's clock states' process noise covariance  $\mathbf{Q}_{\text{clk},r}$ . The number of available BTSs was varied in  $N \in \{6, 8, 10, 12\}$ . The initial speed of the UAV  $v_{r_1}$  was varied in  $v_{r_1} \in \{4, 9, 13\}$  m/s. The process noise covariance of the UAV-mounted receiver's oscillator was varied to correspond to that of a temperature-compensated crystal oscillator (TCXO) and to that of an oven-controlled crystal oscillator (OCXO). Subsequently, 24 cases were simulated. The UAV's position and velocity and the UAV-mounted receiver's and BTSs' clock states were simulated using the dynamics discussed in Sections II-A and II-B with a sampling time  $T = 0.1$  s. The carrier phase measurement to each BTS was simulated according to (1) with  $\lambda = 33.96$  cm, which corresponds to a wavelength dedicated for a cellular CDMA channel. The 3GPP2 protocol requires cellular BTSs to be synchronized within 3  $\mu\text{s}$  to GPS with a

TABLE I  
Simulation Settings

Parameter	Value	Unit
$N$	{6, 8, 10, 12}	-
$\lambda$	0.3396	m
$T$	0.1	s
$\mathbf{R}$	$(0.03) \cdot \mathbf{I}_{N \times N}$	$\text{m}^2$
$\Sigma_{r_r}(j)$	$\begin{bmatrix} 14.36 & -6.97 \\ -6.97 & 11.90 \end{bmatrix}$	$\text{m}^2$
$j$	{0, 1}	-
$\mathbf{r}_r(0)$	$[-500, -1500]^T$	m
$\dot{\mathbf{r}}_r(0)$	$v_{r1} \cdot [0.316, 0.949]^T$	m/s
$v_{r1}$	{4, 9, 13}	m/s
$h_{-2,r}$	{ $h_{-2,\text{TCXO}}, h_{-2,\text{OCXO}}$ }	$\text{s}^{-1}$
$h_{0,r}$	{ $h_{0,\text{TCXO}}, h_{0,\text{OCXO}}$ }	s
$h_{-2,s_n}$	$h_{-2,\text{OCXO}}$	$\text{s}^{-1}$
$h_{0,s_n}$	$h_{0,\text{OCXO}}$	s
$h_{-2,\text{TCXO}}$	$2 \times 10^{-20}$	$\text{s}^{-1}$
$h_{-2,\text{OCXO}}$	$4 \times 10^{-23}$	$\text{s}^{-1}$
$h_{0,\text{TCXO}}$	$2 \times 10^{-19}$	s
$h_{0,\text{OCXO}}$	$8 \times 10^{-20}$	s
$\tilde{q}_x, \tilde{q}_y$	0.03	$\text{m}^2/\text{s}^3$
$c\delta t_r, c\delta t_{s_n}$	$\mathcal{U}(-900, 900)^*$	m
$\dot{c}\delta t_r, \dot{c}\delta t_{s_n}$	$\mathcal{U}(-5, 5)$	m/s
$N_n$	$\mathcal{U}\{-500, 500\}^{**}$	cycles

\* $\mathcal{U}(a, b)$  denotes the probability density function of a continuous uniformly distributed random variable in  $(a, b)$ . \*\*  $\mathcal{U}[c, d]$  denotes the probability mass function of a discrete uniformly distributed random variable in  $[c, d]$ .

frequency stability of 50 ppb [17], [18]. It was assumed that the BTSs are equipped with OCXOs to meet these requirements [42], [43]. The EKF was initialized according to the framework in Section III-D. The simulation settings are summarized in Table I.

### B. Single Realization Simulation Results

A single realization of the EKF errors and associated  $\pm 3\sigma$  bounds for the UAV's position and velocity and the clock error states corresponding to  $c\delta t_1$  and  $\dot{c}\delta t_1$  are plotted in Fig. 2 with  $N = 10$ ,  $v_{r1} = 9$  m/s, and an OCXO-equipped receiver. The decreasing  $\pm 3\sigma$  bounds and converging errors shown in Fig. 2 do not contradict the main results in Section IV-C that the estimation error can be exponentially bounded and bounded with probability 1, as per Definitions IV.1 and IV.2. The remaining clock error states, namely  $c\delta t_n$  and  $\dot{c}\delta t_n$  for  $n = 2, 3, \dots, 9$ , behave similarly to the ones plotted in Fig. 2.

### C. MC Simulation Results

Next, MC simulations were conducted by randomizing the process and measurement noise, as well as the initial UAV position estimates  $\hat{\mathbf{r}}_r(0)$  and  $\hat{\mathbf{r}}_r(1)$ . A total of 200 MC simulations were performed for each case described in

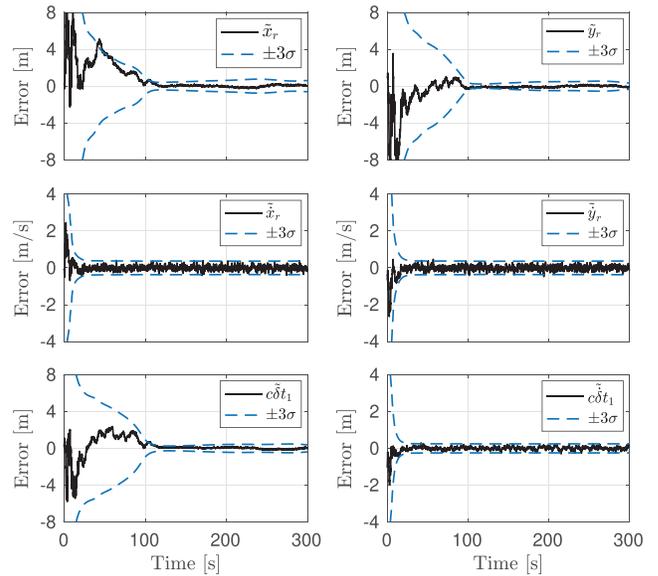


Fig. 2. Simulation results: single realization of EKF errors and associated  $\pm 3\sigma$  bounds for the UAV position and velocity and the clock error states corresponding to  $c\delta t_1$  and  $\dot{c}\delta t_1$  with  $N = 10$ ,  $v_{r1} = 9$  m/s, and an OCXO-equipped receiver.

TABLE II  
MC Simulation Results

		Position RMSE (m)			Final position error (m)		
		4	9	13	4	9	13
TCXO	$N$						
	6	33.47	22.85	13.15	14.28	10.23	7.29
	8	30.34	18.02	9.57	12.03	4.85	0.95
	10	27.96	17.25	9.84	9.69	4.94	1.09
	12	17.31	9.83	6.78	8.72	3.76	0.92
OCXO	6	25.97	14.84	11.18	10.15	9.65	6.52
	8	25.73	15.73	8.97	9.53	4.55	1.03
	10	24.77	13.77	5.50	8.80	2.58	0.48
	12	16.61	9.57	3.64	8.37	4.98	0.33

Section V-A. The total position RMSE and the final position error RMSE are tabulated in Table II.

The following can be deduced from Table II. First, as expected, the estimation performance improves as the number of available BTSs increases. Second, the receiver's clock quality significantly affects the estimation performance for  $N \leq 10$ . For  $N > 10$ , the effect of the receiver's oscillator quality becomes less significant. Third, as the initial speed increases, the estimation performance improves. Faster UAV speeds result in a faster change in the bearing angles between the UAV and the BTSs, yielding an increase in the amount of information coming from cellular carrier phase measurements.

Next, the LB proposed in Section IV-B is studied. To this end, 200 MC simulations were performed with  $N = 10$ ,  $v_{r1} = 9$  m/s, and an OCXO-equipped receiver. The time history of the logarithm of the determinant of the estimation error covariance for each realization, denoted by  $\log_{\det}[\mathbf{P}(k|k)]$ , is plotted in Fig. 3 along with the logarithm

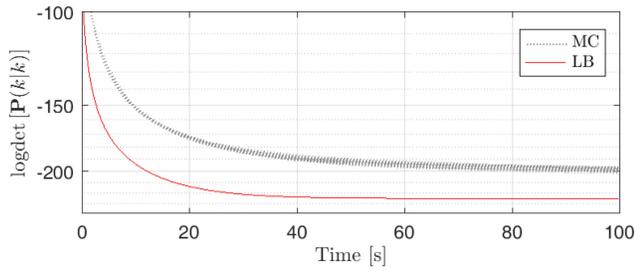


Fig. 3. MC simulation results: 200 realizations of  $\log\det[\mathbf{P}(k|k)]$  along with the logarithm of the determinant of the theoretical LB obtained according to Section IV-B with  $N = 10$ ,  $v_{r_1} = 9$  m/s, and an OCXO-equipped receiver.

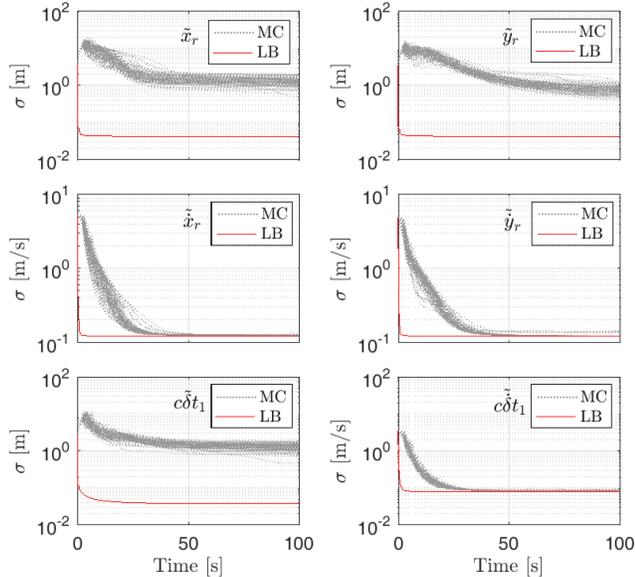


Fig. 4. MC simulation results: 200 realizations of  $1\sigma$  bound for the UAV's position and velocity as well as the clock error states corresponding to  $c\delta t_1$  and  $\delta t_1$ , and the corresponding theoretical LB obtained according to Section IV-B with  $N = 10$ ,  $v_{r_1} = 9$  m/s, and an OCXO-equipped receiver.

of the determinant of the theoretical LB obtained according to Section IV-B. Note that  $\log\det[\mathbf{P}(k|k)]$  is related to the volume of the uncertainty ellipsoid [44]. Moreover, the  $1\sigma$  bound calculated by the EKF is plotted for each MC realization in Fig. 4 for the UAV's position and velocity as well as the clock error states corresponding to  $c\delta t_1$  and  $\delta t_1$ . The theoretical LB calculated using the method proposed in Section IV-B is also plotted. Note that the  $\sigma$  bounds of the remaining clock error states, namely  $c\delta t_n$  and  $\delta t_n$  for  $n = 2, 3, \dots, 10$ , behave similarly to the ones plotted in Fig. 4.

The following can be concluded from these plots. First, the  $\log\det[\mathbf{P}(k|k)]$  plot in Fig. 3 shows that: 1) the estimation error uncertainty is decreasing for all MC realizations and 2) the theoretical LB is not violated. Second, each component of the theoretical LB in Fig. 4 bounds the respective component in the estimation error covariance. This result is not guaranteed to hold by the algorithm to compute the theoretical LB; however, it seems to hold.

Third, the theoretical LB is tight for the velocity and clock drift states, but not tight for the position and clock bias states.

## VI. EXPERIMENTAL RESULTS

In this section, two experiments are conducted demonstrating UAV navigation via the framework developed in this article. In the following experiments, the altitude of the UAVs was known from their on-board navigation system.

### A. Measurement Noise Statistics

The CDMA and LTE receivers employed in the experiments use second-order coherent phase-locked loops (PLLs), for which it can be shown that the measurement noise variance  $\sigma_n^2$  is given by  $\sigma_n^2(k) = \lambda^2 \frac{B_{\text{PLL}}}{C/N_{0n}(k)}$ , where  $B_{\text{PLL}}$  is the receiver's PLL noise equivalent bandwidth and  $C/N_{0n}(k)$  is the  $n$ th BTS's carrier-to-noise ratio measured by the receiver [45]. In the following experiments,  $B_{\text{PLL}}$  was set to 3 Hz.

### B. Hardware and Filter Description

An Autel Robotics X-Star Premium UAV was used for the first experiment and a DJI Matrice 600 was used for the second experiment. In each experiment, the UAVs were equipped with an Ettus E312 universal software radio peripheral (USRP), a consumer-grade 800/1900-MHz cellular antenna, and a small consumer-grade GPS antenna to discipline the on-board oscillator. In both experiments, the UAV-mounted receivers were tuned to listen to cellular signals in the two bands allocated for cellular communication in the U.S.: the 800- and 1900-MHz bands. An E312 USRP was tuned to a 882.75-MHz carrier frequency (i.e.,  $\lambda = 33.96$  cm), which is a cellular CDMA channel allocated for the U.S. cellular provider Verizon Wireless. In the second experiment, the UAV was also equipped with a second antenna and another E312 USRP, which was tuned to a 1955-MHz carrier frequency (i.e.,  $\lambda = 15.33$  cm), which is an LTE channel allocated for the U.S. cellular provider AT&T. Samples of the received signals were stored for offline postprocessing. The cellular carrier phase measurements were given at a rate of 37.5 Hz, i.e.,  $T = 26.67$  ms. The ground-truth reference for each UAV trajectory was taken from its on-board navigation system, which uses GPS, an inertial measurement unit (IMU), and other sensors. The hovering horizontal precision of the UAVs are reported to be 2 m for the X-Star Premium by Autel Robotics and 1.5 m for the Matrice 600 by DJI. The E312 USRPs are equipped with TCXOs with  $h_{0,r} = 2 \times 10^{-19}$  and  $h_{-2,r} = 2 \times 10^{-20}$ , and the BTSs are assumed to be equipped with OCXOs with  $h_{0,s_n} = 8 \times 10^{-20}$  and  $h_{-2,s_n} = 4 \times 10^{-23}$ . The  $x$  and  $y$  continuous-time acceleration noise spectra were set to  $\tilde{q}_x = \tilde{q}_y = 0.03$  m<sup>2</sup>/s<sup>3</sup> for both experiments. The EKF was initialized according to the framework in Section III-D with initial position estimates obtained from the UAVs' on-board

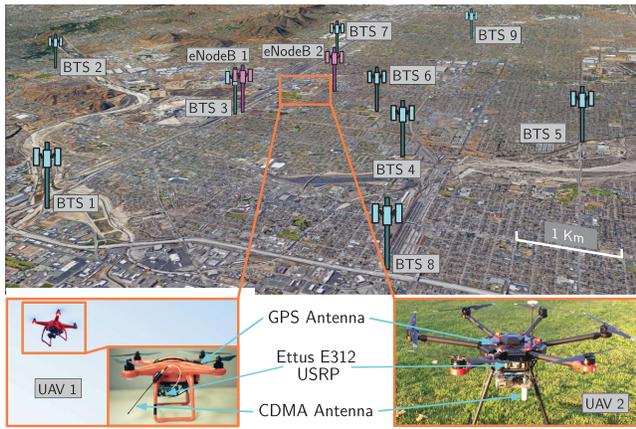


Fig. 5. Experimental setup and BTS and eNodeB layout. The environment consists of nine cellular CDMA BTSs (cyan towers) and two LTE eNodeBs (magenta towers).

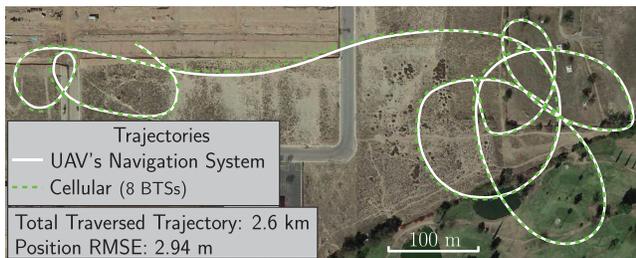


Fig. 6. Experiment 1 ( $N = 8$ ): True UAV trajectory and estimated UAV trajectory via cellular carrier phase measurements with the proposed EKF framework. The true and estimated trajectories are shown in solid and dashed lines, respectively. Map data: Google Earth.

navigation systems. The experimental setup and BTS and eNodeB layout is shown in Fig. 5.

### C. Experiment 1: UAV Navigation Results

In the first experiment, the UAV's total traversed trajectory was 2.6 km, which was completed in 4 min and 40 s. Over the course of the experiment, the UAV-mounted receiver was listening to eight cellular CDMA BTSs, as shown in Fig. 5 (denoted BTSs 1–8). The positions of the BTSs were obtained in two steps: 1) the framework described in [40] was used to obtain an initial map of the cellular BTS locations; then, 2) Google Earth was used to determine the final position of the BTSs. Fig. 6 shows the true and estimated UAV trajectories. The total position RMSE was found to be 2.94 m with a final estimation error at the end of the UAV's flight of 2.23 m. The EKF position error and the associated  $\pm 3\sigma$  bounds as well as the position  $1\sigma$  LB obtained according to Section IV-B are shown in Fig. 7. In order to study the effect of the number of BTSs and their relative geometry, the EKF was run again using BTSs 1 through 4 ( $N = 4$ ) and then BTSs 1 through 6 ( $N = 6$ ). The resulting EKF position errors and the associated  $\pm 3\sigma$  bounds as well as the position  $1\sigma$  theoretical LB obtained according to Section IV-B are shown in Fig. 8. The total position RMSEs and final errors are summarized in Table III.

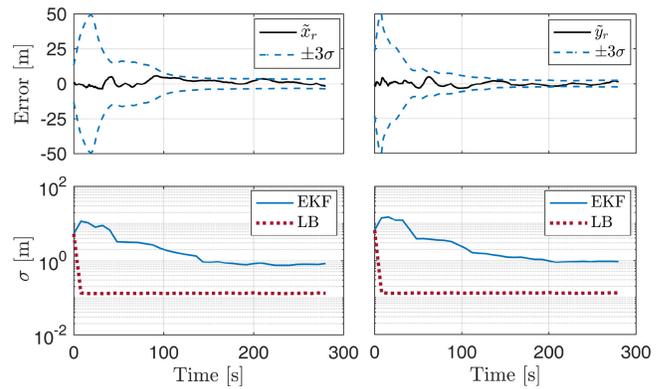


Fig. 7. Experiment 1 ( $N = 8$ ). (Top) UAV's position estimation error trajectories and associated  $\pm 3\sigma$  bounds. (Bottom) position estimation error standard deviations and the theoretical LB obtained according to Section IV-B.

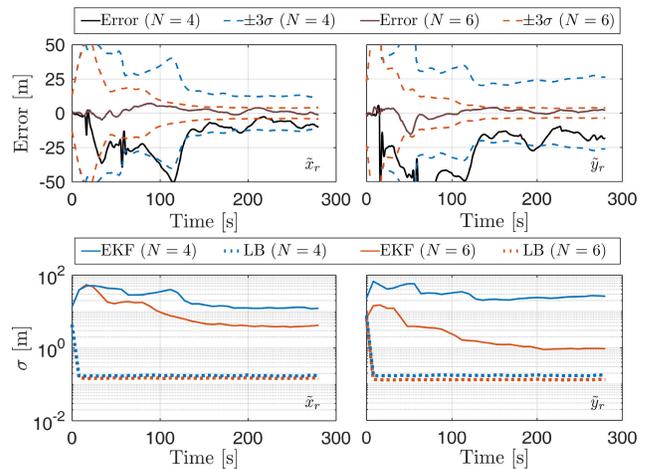


Fig. 8. Experiment 1 ( $N = 4$  and  $N = 6$ ). (Top) UAV's position estimation error trajectories and associated  $\pm 3\sigma$  bounds. (Bottom) position estimation error standard deviations and the theoretical LB obtained according to Section IV-B.

TABLE III  
Experimental Results

$N$	Experiment 1		Experiment 2	
	Position RMSE (m)	Final position error (m)	Position RMSE (m)	Final position error (m)
4	37.39	21.29	30.77	39.81
6	4.68	2.41	14.96	12.84
8	2.94	2.23	-	-
9	-	-	5.99	3.46

### D. Experiment 2: UAV Navigation Results

In the second experiment, the UAV's total traversed trajectory was 2.9 km, which was completed in 5 min. In this experiment, the receiver on-board the UAV was listening to seven cellular CDMA BTSs and two LTE eNodeBs shown in Fig. 5. The BTS and eNodeB positions were determined the same way as in the first experiment. The true and estimated UAV trajectories are shown in Fig. 9. The total position RMSE was found to be 5.99 m with

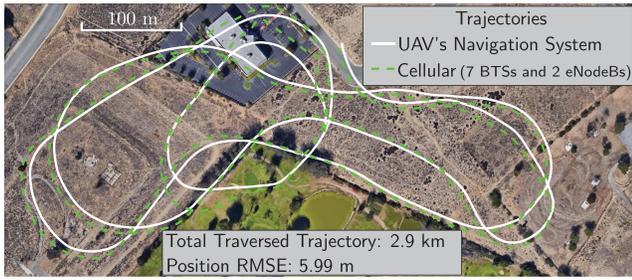


Fig. 9. Experiment 2 ( $N = 9$ ): True UAV trajectory and estimated UAV trajectory via cellular carrier phase measurements with the proposed EKF framework. The true and estimated trajectories are shown in solid and dashed lines, respectively. Map data: Google Earth.

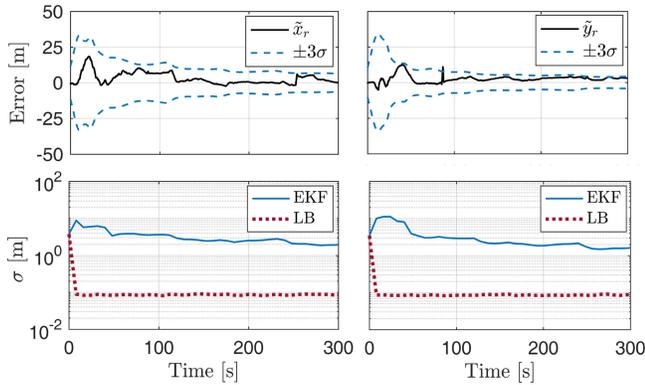


Fig. 10. Experiment 2 ( $N = 9$ ). (Top) UAV's position estimation error trajectories and associated  $\pm 3\sigma$  bounds. (Bottom) position estimation error standard deviations and the theoretical LB obtained according to Section IV-B.

a final estimation error at the end of the UAV's flight of 3.46 m. The EKF position error and the associated  $\pm 3\sigma$  bounds as well as the position  $1\sigma$  LB obtained according to Section IV-B are shown in Fig. 10. In order to study the effect of the number of BTSs and their relative geometry as in the first experiment, the EKF was run again using BTSs 1 through 4 ( $N = 4$ ) and then BTSs 1 through 5 and eNodeB 1 ( $N = 6$ ). The resulting EKF position errors and the associated  $\pm 3\sigma$  bounds as well as the position  $1\sigma$  theoretical LB obtained according to Section IV-B are shown in Fig. 11. The total position RMSEs and final errors are summarized in Table III.

## E. Discussion

First, it is important to note that the favorable geometry of the BTSs and eNodeBs comes by virtue of: 1) the cellular network structure where cells are typically organized in adjacent hexagons with servicing BTSs at the center of each hexagon and 2) diversity of cellular providers. Moreover, since the wireless channel is particularly good for UAVs (line of sight is almost always maintained), it is very likely the UAV will be able to reliably listen to geometrically diverse BTSs, as shown in Fig. 5.

Second, it was shown in Section IV-C that the estimation error can be exponentially bounded and bounded with

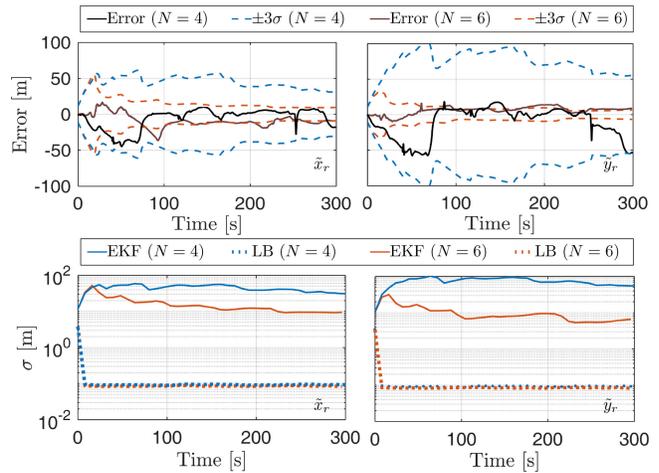


Fig. 11. Experiment 2 ( $N = 4$  and  $N = 6$ ). (Top) UAV's position estimation error trajectories and associated  $\pm 3\sigma$  bounds. (Bottom) position estimation error standard deviations and the theoretical LB obtained according to Section IV-B.

probability 1, as per Definitions IV.1 and IV.2, respectively, and the estimation error covariance is bounded according to (26). None of the experiments contradicts this result, since they both show a decreasing  $\pm 3\sigma$  bounds and converging errors, as shown in Figs. 7, 8, 10, and 11 without violating the proposed LB.

Third, the UAVs in both experiments were flying at almost constant speeds of 9.3 and 9.7 m/s for experiments 1 and 2, respectively. Based on the simulation results presented in Section V, the RMSE and final error are expected to decrease as the UAV speed increases.

Fourth, note that the proposed framework considers imperfect knowledge of the initial state (i.e., it is initialized with an initial estimate and corresponding uncertainty). It is important that the initial estimate be consistent with the initial estimation error covariance (uncertainty). The initialization scheme proposed in Section III-D ensures that the initial error and the initial uncertainty are consistent.

Fifth, the framework studied in this article assumed the BTSs' positions to be known *a priori* with no uncertainty. Having uncertainty in the BTS's position does not guarantee the applicability of Theorem IV.2. Future work could extend this work to the case with unknown/uncertain BTS positions. The reader is encouraged to look at the work in [5] and [26] for more details on how uncertainty in the BTS positions affects the performance of an opportunistic navigation framework.

Sixth, the UAV's on-board oscillator was disciplined using GPS signals during the experiment. The effect of not disciplining the on-board oscillator can be captured by increasing the clock process noise covariance matrix. Based on the results in Section V, the RMSE and the final error are expected to increase slightly when the on-board oscillator is not disciplined by GPS signals. It is important to note that now one has to find a new  $(\epsilon, \delta)$  pair for Theorem V.2 to

hold. Since  $\mathbf{Q}$  increases in a GPS-denied environment, then  $\delta$  is likely to increase. Intuitively, one expects  $\epsilon$  to decrease in this case for Theorem V.2 to hold. More details can be found in [32].

**REMARK 7** The EKF employs statistical models to propagate the position and velocity of the UAV and the clock bias and drift differences. Such models will inherently mismatch the true dynamics of the UAV and clock states, possibly yielding large estimation errors. Using an IMU to propagate the position and velocity states of the UAV should yield better results [5]. Moreover, an adaptive filter may be employed to simultaneously estimate the clock states' process noise covariance to reduce the clock model mismatch [43].

## VII. CONCLUSION

This article presented a framework for UAV navigation with asynchronous cellular signals. The framework employs precise cellular carrier phase measurements and an EKF to estimate the position and velocity of the UAV as well as the difference of the biases and drifts between the receiver's and each BTS's clock. An EKF initialization scheme was also proposed. Moreover, it was shown that: 1) this framework is observable and 2) the EKF error state is asymptotically stable in a mean square sense and bounded with probability 1. An LB for the EKF's estimation error covariance was provided. MC simulations showed that this bound is not violated and studied the performance of the proposed framework for varying number of BTSs, initial UAV speeds, and UAV-mounted receiver clock qualities. It was also shown that the EKF estimation error covariance is upper bounded. Finding such an upper bound is involved and could be the subject of future work. Two sets of experimental results on two different UAVs showed that this framework can achieve 2.94 and 5.99 m position RMSE, over UAV trajectories of 2.6 and 2.9 km, respectively.

## APPENDIX A DERIVATION OF THE ML ESTIMATE FOR EKF INITIALIZATION

This appendix derives  $\hat{\mathbf{x}}_{\text{ML-ini}}$  and  $\mathbf{P}_{\text{ML-ini}}$  used in the EKF initialization. For a sufficiently small  $T$ , the receiver velocity at  $k = 1$  may be expressed as

$$\dot{\mathbf{r}}_r(1) = \frac{1}{T} [\mathbf{r}_r(1) - \mathbf{r}_r(0)] \quad (32)$$

and the  $n$ th BTS's clock drift at  $k = 1$  as

$$c\hat{\delta}t_n(1) \approx \frac{1}{T} [c\delta t_n(1) - c\delta t_n(0)]. \quad (33)$$

From (32) and (33), one may express  $\mathbf{x}(0)$  as

$$\mathbf{x}(0) = \mathbf{F}^{-1}\mathbf{x}(1). \quad (34)$$

Using (4), (7), and (34), the following measurement equation is obtained:

$$\mathbf{z}_{\text{ini}} = \mathbf{h}_{\text{ini}}[\mathbf{x}(1)] + \mathbf{v}_{\text{ini}} \quad (35)$$

where the vector-valued function  $\mathbf{h}_{\text{ini}}$  is given by

$$\mathbf{h}_{\text{ini}}[\mathbf{x}(1)] = \begin{bmatrix} \mathbf{r}_r(1) \\ \mathbf{r}_r(1) - T\dot{\mathbf{r}}_r(1) \\ \mathbf{h}[\mathbf{x}(1)] \\ \mathbf{h}[\mathbf{F}^{-1}\mathbf{x}(1)] \end{bmatrix}$$

and the measurement noise vector is given

$$\mathbf{v}_{\text{ini}} = [\mathbf{v}_{r_r}^T(1), \mathbf{v}_{r_r}^T(0), \mathbf{v}^T(1), \mathbf{v}^T(0)]^T$$

which is a zero-mean white Gaussian random vector with the block-diagonal covariance matrix  $\Sigma_{\text{ini}} = \text{diag}[\Sigma_{r_r}(1), \Sigma_{r_r}(0), \mathbf{R}(1), \mathbf{R}(0)]$ . The ML estimate of  $\mathbf{x}(1)$  can be, therefore, obtained from (35) according to

$$\hat{\mathbf{x}}_{\text{ML-ini}} = \underset{\mathbf{x}(1)}{\text{argmax}} \Lambda[\mathbf{z}_{\text{ini}}; \mathbf{x}(1)] \quad (36)$$

where  $\Lambda[\mathbf{z}_{\text{ini}}; \mathbf{x}(1)]$  is the likelihood function of  $\mathbf{z}_{\text{ini}}$  parameterized by  $\mathbf{x}(1)$ , which is the multivariate Gaussian probability density function with zero-mean and covariance  $\Sigma_{\text{ini}}$ . The maximization problem in (36) is equivalent to

$$\hat{\mathbf{x}}_{\text{ML-ini}} = \underset{\mathbf{x}(1)}{\text{argmin}} \{\mathbf{z}_{\text{ini}} - \mathbf{h}_{\text{ini}}[\mathbf{x}(1)]\}^T \Sigma_{\text{ini}}^{-1} \{\mathbf{z}_{\text{ini}} - \mathbf{h}_{\text{ini}}[\mathbf{x}(1)]\}$$

which can be solved using the Gauss-Newton method. It can be shown that

$$\hat{\mathbf{x}}_{\text{ML-ini}} = \left[ \hat{\mathbf{r}}_{r,\text{ML-ini}}^T, c\hat{\delta}t_{1,\text{ML-ini}}, \dots, c\hat{\delta}t_{N,\text{ML-ini}}, \hat{\mathbf{r}}_{r,\text{ML-ini}}^T, c\hat{\delta}t_{1,\text{ML-ini}}, \dots, c\hat{\delta}t_{N,\text{ML-ini}} \right]^T$$

$$\hat{\mathbf{r}}_{r,\text{ML-ini}} = \mathbf{z}_{r_r}(1) \quad (37)$$

$$\hat{\mathbf{r}}_{r,\text{ML-ini}} = \frac{1}{T} [\mathbf{z}_{r_r}(1) - \mathbf{z}_{r_r}(0)] \quad (38)$$

$$c\hat{\delta}t_{n,\text{ML-ini}} = z_n(1) - d_{r,n}(1) \quad (39)$$

$$c\hat{\delta}t_{n,\text{ML-ini}} = \frac{1}{T} [z_n(1) - z_n(0) + d_{r,n}(0) - d_{r,n}(1)] \quad (40)$$

with  $d_{r,n}(j) \triangleq \|\mathbf{z}_{r_r}(j) - \mathbf{r}_{s_n}\|$  for  $j = 0, 1$ . It can also be shown that the estimation error covariance associated with  $\hat{\mathbf{x}}_{\text{ML-ini}}$  is given by

$$\mathbf{P}_{\text{ML-ini}} = \mathbf{A}_{\text{ini}} \Sigma_{\text{ini}} \mathbf{A}_{\text{ini}}^T$$

$$\mathbf{A}_{\text{ini}} \triangleq \begin{bmatrix} \mathbf{I}_{2 \times 2} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times N} & \mathbf{0}_{2 \times N} \\ -\mathbf{h}_{r,1}^T(1) & \mathbf{0}_{1 \times 2} & \mathbf{e}_{N,1}^T & \mathbf{0}_{1 \times N} \\ \vdots & \vdots & \vdots & \vdots \\ -\mathbf{h}_{r,N}^T(1) & \mathbf{0}_{1 \times 2} & \mathbf{e}_{N,N}^T & \mathbf{0}_{1 \times N} \\ \frac{1}{T} \mathbf{I}_{2 \times 2} & -\frac{1}{T} \mathbf{I}_{2 \times 2} & \mathbf{0}_{2 \times N} & \mathbf{0}_{2 \times N} \\ -\frac{1}{T} \mathbf{h}_{r,1}^T(1) & \frac{1}{T} \mathbf{h}_{r,1}^T(0) & \frac{1}{T} \mathbf{e}_{N,1}^T & -\frac{1}{T} \mathbf{e}_{N,1}^T \\ \vdots & \vdots & \vdots & \vdots \\ -\frac{1}{T} \mathbf{h}_{r,N}^T(1) & \frac{1}{T} \mathbf{h}_{r,N}^T(0) & \frac{1}{T} \mathbf{e}_{N,N}^T & -\frac{1}{T} \mathbf{e}_{N,N}^T \end{bmatrix} \quad (41)$$

with  $\mathbf{h}_{r,n}(j) \triangleq \frac{z_{r,n}(j) - r_{s_n}}{d_{r,n}(j)}$  for  $j = 0, 1$ , and  $\mathbf{e}_{N,n} \in \mathbb{R}^N$  is the standard basis vector consisting of a one at the  $n$ th element and zeros elsewhere.

## APPENDIX B DERIVATION OF THE UPPER BOUNDS IN (8)

In this appendix, the estimation error covariance associated with the initial speed estimate  $\hat{v}_{r_1} \triangleq \|\hat{\mathbf{r}}_{r,\text{ML,ini}}\|$  and the initial velocity direction unit vector estimate  $\hat{\mathbf{u}}_{r_1} \triangleq \frac{\hat{\mathbf{r}}_{r,\text{ML,ini}}}{\|\hat{\mathbf{r}}_{r,\text{ML,ini}}\|}$  are studied. Let  $v_{r_1} \triangleq \|\dot{\mathbf{r}}_r(1)\|$  denote the true initial speed and  $\mathbf{u}_{r_1} \triangleq \frac{\dot{\mathbf{r}}_r}{\|\dot{\mathbf{r}}_r\|}$  denote the true initial direction unit vector. Using first-order Taylor series expansion around  $\dot{\mathbf{r}}_r(1)$ , the initial speed estimate may be approximated with

$$\hat{v}_{r_1} \approx v_{r_1} - \mathbf{u}_{r_1}^T \tilde{\mathbf{r}}_{r,\text{ini}} \quad (42)$$

where  $\tilde{\mathbf{r}}_{r,\text{ini}} \triangleq \dot{\mathbf{r}}_r(1) - \hat{\mathbf{r}}_{r,\text{ML,ini}}$ . The initial speed estimation error can be calculated from (42) as

$$\tilde{v}_{r_1} \triangleq v_{r_1} - \hat{v}_{r_1} = \mathbf{u}_{r_1}^T \tilde{\mathbf{r}}_{r,\text{ini}}$$

and its associated estimation error variance may be obtained according

$$\sigma_{v_{r_1}}^2 \triangleq \mathbb{E}[\tilde{v}_{r_1}^2] = \mathbf{u}_{r_1}^T \mathbb{E}[\tilde{\mathbf{r}}_{r,\text{ini}} \tilde{\mathbf{r}}_{r,\text{ini}}^T] \mathbf{u}_{r_1}.$$

It can be shown from (38) and (41) that

$$\mathbf{P}_{\tilde{\mathbf{r}}_{r,\text{ini}}} \triangleq \mathbb{E}[\tilde{\mathbf{r}}_{r,\text{ini}} \tilde{\mathbf{r}}_{r,\text{ini}}^T] = \frac{1}{T^2} \bar{\Sigma}_{r_r}$$

where  $\bar{\Sigma}_{r_r} \triangleq \Sigma_{r_r}(0) + \Sigma_{r_r}(1)$ . Since  $\mathbf{u}_{r_1}$  is a unit vector,  $\sigma_{v_{r_1}}^2$  may be bounded according to

$$\sigma_{v_{r_1}}^2 \leq \frac{\lambda_{\max,\dot{r}}}{T^2} \quad (43)$$

where  $\lambda_{\max,\dot{r}}$  denotes the maximum eigenvalue of  $\bar{\Sigma}_{r_r}$ . Note that since  $\bar{\Sigma}_{r_r}$  is positive definite, then  $\lambda_{\max,\dot{r}}$  is a positive real number that only depends on  $\bar{\Sigma}_{r_r}$ .

Next, the estimation error covariance of  $\hat{\mathbf{u}}_{r_1}$  is characterized. Using first-order Taylor series expansion around  $\dot{\mathbf{r}}_r$ ,  $\hat{\mathbf{u}}_{r_1}$  may be approximated with

$$\hat{\mathbf{u}}_{r_1} \approx \mathbf{u}_{r_1} - \Psi \tilde{\mathbf{r}}_{r,\text{ini}} \quad (44)$$

where  $\Psi \triangleq \frac{1}{v_{r_1}} (\mathbf{I}_{2 \times 2} - \mathbf{u}_{r_1} \mathbf{u}_{r_1}^T)$ . The initial direction unit vector estimation error can be calculated from (44) as

$$\tilde{\mathbf{u}}_{r_1} \triangleq \mathbf{u}_{r_1} - \hat{\mathbf{u}}_{r_1} = \Psi \tilde{\mathbf{r}}_{r,\text{ini}}$$

The matrix  $\Psi$  is an annihilator matrix; therefore,  $\|\Psi\| = 1$ . The estimation error covariance associated with  $\tilde{\mathbf{u}}_{r_1}$  is given by

$$\mathbf{P}_{\tilde{\mathbf{u}}_{r_1}} \triangleq \mathbb{E}[\tilde{\mathbf{u}}_{r_1} \tilde{\mathbf{u}}_{r_1}^T] = \frac{1}{v_{r_1}^2} \Psi \bar{\Sigma}_{r_r} \Psi^T. \quad (45)$$

Using (32) and (45),  $\|\mathbf{P}_{\tilde{\mathbf{u}}_{r_1}}\|$  may be bounded according to

$$\sigma_{\tilde{\mathbf{u}}_{r_1}}^2 \triangleq \|\mathbf{P}_{\tilde{\mathbf{u}}_{r_1}}\| \leq \frac{\lambda_{\max,\dot{r}}}{\|\dot{\mathbf{r}}_r(1) - \dot{\mathbf{r}}_r(0)\|^2}. \quad (46)$$

## APPENDIX C THEORETICAL BACKGROUND ON OBSERVABILITY AND EKF ESTIMATION ERROR BOUNDS

This appendix provides relevant theoretical background on observability and EKF estimation error bounds.

### A. OBSERVABILITY OF LINEAR AND NONLINEAR SYSTEMS

Consider the discrete-time linear time-varying system

$$\mathbf{x}(k+1) = \mathbf{F}(k)\mathbf{x}(k) + \mathbf{\Gamma}(k)\mathbf{u}(k) \quad (47)$$

$$\mathbf{y}(k) = \mathbf{H}(k)\mathbf{x}(k) \quad (48)$$

where  $\mathbf{x} \in \mathbb{R}^{n_x}$  is the system's state,  $\mathbf{u} \in \mathbb{R}^{n_u}$  is the input, and  $\mathbf{y} \in \mathbb{R}^{n_y}$  is the measurement.

Observability of the discrete-time linear time-varying system defined in (47) and (48) is usually determined by studying the rank of either the observability Gramian or the observability matrix. The following theorem from [46] states a necessary and sufficient condition for observability of linear time-varying systems through the  $l$ -step observability matrix.

**THEOREM C.1** The discrete-time linear time-varying system defined in (47) and (48) is  $l$ -step observable if and only if the  $l$ -step observability matrix, defined as

$$\mathcal{O}(k, k+l) \triangleq \begin{bmatrix} \mathbf{H}(k) \\ \mathbf{H}(k+1)\Phi(k+1, k) \\ \vdots \\ \mathbf{H}(k+l-1)\Phi(k+l-1, k) \end{bmatrix} \quad (49)$$

is full rank, i.e.,  $\text{rank}[\mathcal{O}(k, k+l)] = n_x$ . The matrix function  $\Phi(k, j)$  is the discrete-time state transition matrix, which is defined as

$$\Phi(k, j) \triangleq \begin{cases} \mathbf{F}(k-1)\mathbf{F}(k-2)\cdots\mathbf{F}(j), & k \geq j+1 \\ \mathbf{I}, & k = j. \end{cases}$$

Linear observability tools may be applied to nonlinear systems by linearizing the dynamics and measurements to obtain  $\mathbf{F}(k)$ ,  $\mathbf{\Gamma}(k)$ , and  $\mathbf{H}(k)$  [31]. The observability results in such case are only valid locally.

### B. EKF ERROR BOUNDS

The following useful lemma from [47] establishes bounds on the KF's estimation error covariance.

**LEMMA C.1** Consider the discrete-time linear time-varying stochastic system

$$\mathbf{x}(k+1) = \mathbf{F}(k)\mathbf{x}(k) + \mathbf{\Gamma}(k)\mathbf{u}(k) + \mathbf{w}(k) \quad (50)$$

$$\mathbf{z}(k) = \mathbf{H}(k)\mathbf{x}(k) + \mathbf{v}(k) \quad (51)$$

where  $\mathbf{x} \in \mathbb{R}^{n_x}$  is the system's state,  $\mathbf{u} \in \mathbb{R}^{n_u}$  is the input,  $\mathbf{w} \in \mathbb{R}^{n_x}$  is a zero-mean white sequence with covariance  $\mathbf{Q}(k)$ ,  $\mathbf{z} \in \mathbb{R}^{n_z}$  is the measurement, and  $\mathbf{v} \in \mathbb{R}^{n_z}$  is a zero-mean white sequence with covariance  $\mathbf{R}(k)$ . Assume that  $\mathbf{w}(k)$  and  $\mathbf{v}(j)$  are uncorrelated for all  $k$  and  $j$ . Let

$\mathbf{P}(k+1|k)$  be a solution to the matrix Riccati difference equation in the KF estimating the state of system (50), (51) given by

$$\begin{aligned} \mathbf{P}(k+1|k) &= \mathbf{F}(k) \{ \mathbf{P}(k|k-1) - \mathbf{P}(k|k-1)\mathbf{H}^\top(k) \cdot \\ &\quad [\mathbf{H}(k)\mathbf{P}(k|k-1)\mathbf{H}^\top(k) + \mathbf{R}(k)]^{-1} \cdot \\ &\quad \mathbf{H}(k)\mathbf{P}(k|k-1) \} \mathbf{F}^\top(k) + \mathbf{Q}(k). \end{aligned}$$

Let the following hold.

- 1) There are real numbers  $q, \bar{q}, \underline{r}, \bar{r} > 0$  such that  $\mathbf{Q}(k)$  and  $\mathbf{R}(k)$  are bounded by

$$\underline{q}\mathbf{I} \leq \mathbf{Q}(k) \leq \bar{q}\mathbf{I}, \quad \underline{r}\mathbf{I} \leq \mathbf{R}(k) \leq \bar{r}\mathbf{I}.$$

- 2) The matrices  $\mathbf{F}(k)$  and  $\mathbf{H}(k)$  satisfy the uniform observability condition.
- 3) The initial condition  $\mathbf{P}(1|0)$  of the matrix Riccati difference equation in the KF is positive definite.

Then, there are real numbers  $\underline{p}, \bar{p} > 0$  such that  $\mathbf{P}(k+1|k)$  is bounded via

$$\underline{p}\mathbf{I} \leq \mathbf{P}(k+1|k) \leq \bar{p}\mathbf{I} \quad \forall k > 0.$$

Next, theoretical background on EKF estimation error bounds is provided. Consider the discrete-time nonlinear stochastic system

$$\mathbf{x}(k+1) = \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k)] + \mathbf{w}(k) \quad (52)$$

$$\mathbf{z}(k) = \mathbf{h}[\mathbf{x}(k)] + \mathbf{v}(k) \quad (53)$$

where  $\mathbf{x} \in \mathbb{R}^{n_x}$  is the system's state,  $\mathbf{u} \in \mathbb{R}^{n_u}$  is the input,  $\mathbf{w} \in \mathbb{R}^{n_x}$  is a zero-mean white sequence with covariance  $\mathbf{Q}(k)$ ,  $\mathbf{z} \in \mathbb{R}^{n_z}$  is the measurement, and  $\mathbf{v} \in \mathbb{R}^{n_z}$  is a zero-mean white sequence with covariance  $\mathbf{R}(k)$ .

An EKF is employed to estimate  $\mathbf{x}(k)$ . Define the EKF linearization errors

$$\begin{aligned} \boldsymbol{\varphi}(k) &\triangleq \mathbf{f}[\mathbf{x}(k), \mathbf{u}(k)] - \mathbf{f}[\hat{\mathbf{x}}(k|k), \mathbf{u}(k)] \\ &\quad - \mathbf{F}(k)[\mathbf{x}(k) - \hat{\mathbf{x}}(k|k)] \end{aligned} \quad (54)$$

$$\begin{aligned} \boldsymbol{\chi}(k) &\triangleq \mathbf{h}[\mathbf{x}(k)] - \mathbf{h}[\hat{\mathbf{x}}(k+1|k)] \\ &\quad - \mathbf{H}(k)[\mathbf{x}(k) - \hat{\mathbf{x}}(k+1|k)] \end{aligned} \quad (55)$$

where  $\mathbf{F}(k)$  and  $\mathbf{H}(k)$  are the dynamics and observation Jacobians, respectively, evaluated at  $\hat{\mathbf{x}}(k|k)$  and  $\hat{\mathbf{x}}(k+1|k)$ , respectively. The main theorem from [32] that establishes conditions for the boundedness of  $\tilde{\mathbf{x}}(k|k)$  is stated.

**THEOREM VII.2** Consider the system defined in (52) and (53) and consider an EKF estimating its state vector. Moreover, let the following assumptions hold.

- 1) There are positive real numbers  $\bar{f}, \bar{h}, \underline{p}, \bar{p}, \underline{q}, \underline{r} > 0$  such that the following bounds hold for every  $k > 0$

$$\|\mathbf{F}(k)\| \leq \bar{f} \quad (56)$$

$$\|\mathbf{H}(k)\| \leq \bar{h} \quad (57)$$

$$\underline{p}\mathbf{I} \leq \mathbf{P}(k+1|k) \leq \bar{p}\mathbf{I} \quad (58)$$

$$\underline{q}\mathbf{I} \leq \mathbf{Q}(k) \quad (59)$$

$$\underline{r}\mathbf{I} \leq \mathbf{R}(k). \quad (60)$$

- 2) The matrix  $\mathbf{F}(k)$  is nonsingular for every  $k > 0$ .
- 3) There are positive real numbers  $\epsilon_\varphi, \epsilon_\chi, \kappa_\varphi, \kappa_\chi > 0$  such that the nonlinear functions  $\boldsymbol{\varphi}(k)$  and  $\boldsymbol{\chi}(k)$  are bounded via

$$\|\boldsymbol{\varphi}(k)\| \leq \kappa_\varphi \|\tilde{\mathbf{x}}(k|k)\|^2 \quad (61)$$

$$\|\boldsymbol{\chi}(k)\| \leq \kappa_\chi \|\tilde{\mathbf{x}}(k|k)\|^2 \quad (62)$$

with  $\|\tilde{\mathbf{x}}(k|k)\| \leq \epsilon_\varphi$  and  $\|\tilde{\mathbf{x}}(k|k)\| \leq \epsilon_\chi$ .

Then, the estimation error  $\tilde{\mathbf{x}}(k|k)$  is exponentially bounded in mean square and bounded with probability 1 as per Definitions IV.1 and IV.2, respectively, provided that the initial estimation error satisfies

$$\|\tilde{\mathbf{x}}(1|1)\| \leq \epsilon \quad (63)$$

and that the covariance matrices of the noise terms are bounded via

$$\mathbf{Q}(k) \leq \delta\mathbf{I}, \quad \mathbf{R}(k) \leq \delta\mathbf{I} \quad (64)$$

for some  $\epsilon, \delta > 0$ .

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**JOE KHALIFE** , Student Member, IEEE  
University of California, Irvine, USA

**ZAHER M. KASSAS** , Senior Member, IEEE  
University of California, Irvine, USA

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