Ephemeris Error Correction for Tracking Non-Cooperative LEO Satellites with Pseudorange Measurements

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Abstract—Correcting the ephemeris of a poorly known “non-cooperative” low Earth orbit (LEO) satellite, by utilizing pseudorange measurements by a ground-based receiver is considered. First, the satellite’s ephemeris error is analyzed, leading to a model parametrizing the error dynamics of the satellite’s argument of latitude. This model is then exploited in an initial tracking stage to reduce the satellite’s large initial along-track errors, after which the position and velocity states of the satellite are estimated. Next, the extended Kalman filter (EKF) is formulated to estimate the satellite’s dynamic states (position, velocity, and clock errors) using pseudorange measurements. Simulation studies comprising varying age-of-ephemeris scenarios are conducted to evaluate the tracking performance of the proposed framework for four different LEO satellite constellations: Starlink, OneWeb, Orbcomm, and Iridium, showing significant reduction in the satellite tracking error via the proposed framework. Experimental results are presented where the proposed framework was used to track 4 Starlink, 2 OneWeb, and 1 Orbcomm LEO satellites using carrier phase measurements. The corrected ephemerides are used to localize a ground receiver. Starting with an initial error of about 2.83 km, using the corrected ephemerides, the final three-dimensional (3-D) localization error was 63.04 m. In contrast, employing the open-loop SGP4-propagated ephemerides resulted in a localization error of about 2.21 km.

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1. INTRODUCTION
The past couple of decades has seen a major shift towards low Earth orbit (LEO) satellite-based communications and sensing. Orbcomm [1] and Myriota [2] provide wide-ranging Internet of Things (IoT) solutions; Iceye [3] specializes in Earth imaging; while Iridium [4] offers low-latency narrowband communication. These LEO constellations have been joined over the past few years with the OneWeb, Starlink, and Kuiper megaconstellations, which aim to offer broadband Internet connectivity [5, 6]. The birth of LEO megeconstellations is envisioned to usher a new era of satellite-based positioning, navigation, and timing (PNT) [7–13].

The growing interest in LEO satellite-based PNT is due to their desirable attributes: (i) abundance and geometric diversity, (ii) high received signal power, (iii) high orbital velocity, and (iv) spectral diversity. These qualities present LEO-PNT systems as a complement or even an alternative to traditional global navigation satellite systems (GNSS) that reside in medium Earth orbit (MEO), and whose signals are vulnerable to attenuation, interference, and cyber attacks [14, 15]. LEO-PNT approaches can be classified into three categories: (i) PNT-dedicated LEO constellations with optimized coverage and performance to deliver ubiquitous and precise PNT services [16–19]; (ii) dual-purposing communication LEO satellites to support the transmission of PNT signals [20–23]; and (iii) exploiting non-positioning LEO signals of opportunity to extract PNT measurements, e.g., Doppler, pseudorange, carrier phase, and angle of arrival, [24–27].

The latter approach, namely opportunistic navigation with LEO satellites, assumes the satellites to be non-cooperative: their ephemerides are poorly known and their signals are unknown at the user (receiver) end. Recent designs of specialized receivers leveraged the periodicity in LEO space vehicles (SV) signals [28–32] or adopted blind frameworks (cognitive receivers) [33–37] to estimate key signal parameters and consequently generate navigation observables. In contrast to GNSS SVs that transmit ephemerides data and clock corrections in their navigation message, most LEO SVs are operated by private companies that do not publicly share information about the satellite’s position, velocity, and time. To address this challenge, the simultaneous tracking and navigation (STAN) framework has been proposed, in which a vehicle-mounted receiver estimates its own states simultaneously with the LEO SVs’ states [38]. The benefits of incorporating differential corrections from a known base for LEO-PNT have been also recently studied [39–41].

High-fidelity precise orbit determination (POD) methods implement numerical propagators that can yield ephemerides with an accuracy on the order of tens-of-meters, with most of the error concentrated in the satellite’s direction of motion, i.e., along-track axis [42]. However, POD methods require rigorous initial conditions including an accurate initial estimate of the satellite’s states and sufficient knowledge about the parameters of various force models, such as atmospheric drag and solar radiation pressure [43]. Recent literature presented machine learning approaches for orbit determination [44–46]. Despite showing great promise, these data-driven techniques do not provide formal assured performance.
A common approach for LEO satellite orbit determination is the notable simplified general perturbation 4 (SGP4) software [47]. SGP4 consists of an analytical orbit propagator that is compatible with two-line element (TLE) files that are used to initialize the propagator. TLE files are published and updated daily by the North American Aerospace Defense Command (NORAD); the first line consists of designation, epoch time, and atmospheric drag parameter data, while the second line lists the SV Keplerian orbital elements: inclination, right ascension of ascending node, eccentricity, argument of perigee, mean anomaly, and mean motion [48]. The TLE-initialized SGP4 propagation scheme has been shown to exhibit ephemeris errors of around 1 to 10 km, 24 hours after a TLE file is updated [49]. These errors can arise from both the initial conditions and the propagation algorithm.

On one hand, TLE data describing LEO satellite orbits may have inherent errors. The calculation of Keplerian elements from the satellite’s position and velocity vectors may cause practical and numerical issues, specifically in the presence of singular orbital elements. This problem may arise for LEO satellites that are typically deployed in near-circular orbits where the eccentricity is nearly zero which may lead to errors in the computation of the argument of perigee [43]. While previous work has developed LEO broadcast ephemeris designs with non-singular element sets [50–52], TLE data remains a readily accessible source for satellite orbit parameters. On the other hand, similarly to most orbit propagators, SGP4 involves dynamical models for the various forces acting on a satellite, including gravitational forces, atmospheric drag, and solar radiation pressure. However, orbit propagation through SGP4 has been shown to exhibit errors concentrated along the satellite’s direction of motion [53]. Specifically, it was found in [54] that SGP4 propagation induces a linearly increasing error in the satellite’s argument of latitude orbital element.

An initial study to model the ephemeris error in opportunistic LEO satellite tracking with pseudorange and Doppler measurements was conducted in [55]. This paper builds on [55] and presents the following additional contributions. First, a nonlinear dynamical model for the satellite’s argument of latitude orbital element based on Gauss’ variational equations is incorporated into the tracking framework. Second, the argument of latitude estimation error covariance mapping is developed to transform the uncertainty into the Cartesian satellite position state-space. Third, extensive simulations are conducted to evaluate the proposed tracking framework’s performance for different LEO satellite constellations and different age-of-ephemeris. The simulations considered a known stationary tracking receiver extracting pseudorange measurements from four satellite constellations: Orbcomm, Starlink, OneWeb, and Iridium. The simulation results show a significant reduction in satellite position errors when the proposed tracking framework is employed as compared to standard open-loop SGP4 propagation and even lower errors when compared to direct closed-loop tracking of the satellite’s Cartesian position and velocity states. Fourth, experimental results are presented demonstrating the efficacy of the LEO SV tracking framework with carrier phase observables that were opportunistically extracted from multi-constellation LEO satellites, namely 4 Starlink, 1 Orbcomm, and 2 OneWeb SVs. The refined ephemerides sets are incorporated in a positioning EKF, showing a reduction in the receiver’s initial horizontal error from 2,828 m to 63.04 m. In contrast, it is shown that incorporating the open-loop SGP4-propagated ephemerides reduces the error to 2,197 m, but, alarmingly, with an inconsistent uncertainty bound.

The rest of the paper is organized as follows. Section 2 describes the dynamics and measurement models. Section 3 presents the proposed LEO SV tracking framework. Section 4 evaluates the performance of the proposed framework via numerical simulations. Section 5 gives experimental results of unknown receiver localization using the proposed corrected ephemerides. Section 6 gives concluding remarks.

2. MODEL DESCRIPTION

LEO Satellite Dynamics

The orbital motion of a LEO satellite is mainly governed by the force exerted by Earth’s gravitational field. This force is represented by the two-body model which describes the satellite’s equations of motion by

\[
\dot{\mathbf{r}}_\text{SV} = \frac{\partial U}{\partial \mathbf{r}_\text{SV}} + \mathbf{\dot{w}}_\text{SV},
\]

where \( \mathbf{r}_\text{SV} \triangleq [x_\text{SV}, \ y_\text{SV}, \ z_\text{SV}]^T \) is the satellite’s position vector in the Earth-centered inertial (ECI) reference frame, \( U \) is the non-uniform gravity potential of Earth, and \( \mathbf{\dot{w}}_\text{SV} \) is a three-dimensional (3-D) vector of acceleration perturbations. Such unmodeled perturbing accelerations include Earth’s non-uniform gravitational potential, atmospheric drag, solar radiation pressure, solar and lunar gravitational attraction, relativistic effects, and solid Earth tides [43].

For a satellite in LEO, it is important to account for the Earth’s oblateness which results in a non-uniform gravitational field. The geopotential can be modeled through the expansion of spherical harmonics that require zonal, sectoral, and tesseral coefficients. The JGM-3 model developed by the Goddard Space Flight Center [56] is commonly used and will be employed in this work while neglecting the tesseral and sectoral terms as they are several orders of magnitude smaller than the zonal terms. The Earth’s zonal harmonics are dominated by the \( J_2 \) term, as all other terms are three orders of magnitude smaller [57]. The equations of motion are then derived by taking the partial derivatives of (1) with respect to the components of \( \mathbf{r}_\text{SV} \), namely \( x_\text{SV}, \ y_\text{SV}, \) and \( z_\text{SV} \), yielding the equations of motion of \( \mathbf{r}_\text{SV} \) given by

\[
\dot{x}_\text{SV} = - \frac{\mu x_\text{SV}}{\|\mathbf{r}_\text{SV}\|^3} \left[ 1 + J_2 \frac{3}{2} \left( \frac{R_E}{\|\mathbf{r}_\text{SV}\|} \right)^2 \left( 1 - 5 \frac{z_\text{SV}^2}{\|\mathbf{r}_\text{SV}\|^2} \right) \right] + \hat{w}_x
\]

\[
\dot{y}_\text{SV} = - \frac{\mu y_\text{SV}}{\|\mathbf{r}_\text{SV}\|^3} \left[ 1 + J_2 \frac{3}{2} \left( \frac{R_E}{\|\mathbf{r}_\text{SV}\|} \right)^2 \left( 1 - 5 \frac{z_\text{SV}^2}{\|\mathbf{r}_\text{SV}\|^2} \right) \right] + \hat{w}_y
\]

\[
\dot{z}_\text{SV} = - \frac{\mu z_\text{SV}}{\|\mathbf{r}_\text{SV}\|^3} \left[ 1 + J_2 \frac{3}{2} \left( \frac{R_E}{\|\mathbf{r}_\text{SV}\|} \right)^2 \left( 3 - 5 \frac{z_\text{SV}}{\|\mathbf{r}_\text{SV}\|} \right) \right] + \hat{w}_z
\]

where \( \mu \) is the Earth’s standard gravitational parameter, \( R_E \) is the mean radius of the Earth, and \( \mathbf{\dot{w}}_\text{SV} \triangleq [\hat{w}_x, \hat{w}_y, \hat{w}_z]^T \) represents the unmodeled acceleration perturbations.

Satellite Orbital Elements

In the previous subsection, the satellite dynamics were described in a Cartesian coordinate system, namely the ECI reference frame. The satellite’s position in space can also be represented in terms of the six orbital elements: semi-major axis \( a \), eccentricity \( e \), inclination \( i \), right ascension \( \Omega \), argument of perigee \( \omega \), and mean anomaly \( M \).
time derivatives of the orbital elements are often de-

The time derivative of motion is calculated as

\[
\frac{d\omega}{dt} = \frac{3J_2\mu R_E^2}{2ehr^3} \left[ \frac{h^2}{\mu r} \cos \nu (1 - 3\sin^2 h \sin^2 u) \\
- (2 + e \cos \nu) \sin(2u) \sin \nu + 2e \cos^2 \sin^2 u \right] \\
+ (2 + e \cos \nu) \sin(2u) \sin^2 \sin^2 u, 
\]

respectively. Consequently, the time derivative of the argument of latitude is calculated as

\[
\frac{du}{dt} = \frac{d\omega}{dt} + \frac{dv}{dt} = \frac{h}{r^2} + \frac{3J_2\mu R_E^2}{2ehr^3} (2e \cos^2 \sin^2 u). 
\] (6)

Measurement Model

It is assumed that the LEO satellite tracker is equipped with a specialized receiver, capable of extracting navigation observables, such as pseudorange or Doppler, from LEO satellite downlink signals. The observables are expressed at time-step \( k \), which represents discrete-time at \( t_k = kT + t_0 \), an initial time \( t_0 \) and sampling time \( T \). The pseudorange measurement \( \rho \) between the receiver and the \( l \)-th LEO SV is modeled as

\[
\rho_l(k) = \| r_r(k) - r_{SV,l}(k) \|_2 + c \cdot [\delta t_r(k) - \delta t_{SV,l}(k)] + \delta_{\text{TOF}}(k) + \delta_{\text{trop},l}(k) + \delta_{\text{iono},l}(k) + \nu_{\rho,l}(k), 
\]

where \( k' \) represents discrete-time at \( t_{k'} = kT + t_0 - \delta_{\text{TOF}} \), with \( \delta_{\text{TOF}} \) being the true time-of-flight (TOF) of the signal from the \( l \)-th LEO satellite; \( r_r \) and \( r_{SV,l} \) are the 3-D position vectors of the receiver and the \( l \)-th LEO SV in the Earth-centered Earth-fixed (ECEF) reference frame; \( c \) is the speed-of-light; \( \delta t_r \) and \( \delta t_{SV,l} \) are the clock biases of the receiver and the \( l \)-th LEO SV, respectively; \( \delta_{\text{trop},l} \) and \( \delta_{\text{iono},l} \) are the ionospheric and tropospheric delays, respectively, affecting the \( l \)-th LEO satellite’s signal; and \( \nu_{\rho,l} \) is the pseudorange measurement noise, which is modeled as a discrete-time zero-mean white Gaussian sequence with variance \( \sigma_{\rho,l} \).

3. Framework Formulation

The large errors of the SGP4-propagated ephemerides can be significantly reduced by estimating the argument of latitude since most of these errors are concentrated in the along-track direction. The framework is based on a two-step process: (i) initialization error reduction and (ii) closed-loop tracking of the SV position and velocity states. The satellite’s position and velocity states are initialized from the SGP4-propagated SV position and velocity at the initial time of visibility to the tracking receiver \( t_0 \). The initial position and velocity estimates are converted from the ECI frame to the orbital element representation to initialize the argument of latitude estimate. The closed-loop tracking EKFs are described next.

Argument of Latitude Tracking Mode—The argument of latitude tracking EKF estimates the state vector \( x = [u, \Delta t_r, \Delta \hat{t}]^T \), where \( \Delta t_r = \delta t_r - \delta t_{SV} \) is the difference between the receiver’s and the LEO SV’s clock biases and \( \Delta \hat{t} = \delta t_r - \delta t_{SV} \) is the difference between the receiver’s
and the LEO SV’s clock drifts. The filter’s time update is performed by numerical integration of (6) and the clock dynamics described in [59], and the pseudorange measurements are used to update the state vector.

The pseudorange measurement Jacobian matrix $H_p$ is formed according to $H_p(k + 1) = [h_{u,p}(k + 1), 1, 0]$ where $h_u$ represents the linearization of $p(k + 1)$ with respect to $\dot{u}(k + 1|k)$ such that

$$h_{u,p}(k + 1) \triangleq \frac{\partial [\rho_i(f + 1), \Delta \delta t, \Delta \hat{\delta} t]}{\partial \rho_{i}\dot{\mathbf{x}}_{SV}(k + 1|k)} \frac{\partial \dot{\mathbf{x}}_{SV}(k + 1|k)}{\partial \dot{u}(k + 1|k)} = -TR_{SV}(k + 1)\left[R_{SV}(k + 1) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}ight],$$

where $I_{SV}(k + 1) \triangleq \frac{\partial [\rho_i(f + 1) - \rho_i(f + 1)]}{\partial \rho_{i}(f + 1)}$ is a unit line-of-sight vector from the receiver to the LEO SV and $R_{SV}(k + 1)$ is the rotation matrix from the ECI $\{i\}$ to the ECEF $\{e\}$ frame of reference that takes into account Earth’s rotation, precession and nutation effects, and polar motion.

State Transition—The state transition from the argument of latitude tracking mode to the position and velocity tracking mode at the end of the initial phase, denoted by the instant $t_i$, is developed as follows. The initial state vector $\mathbf{x} = [u, \Delta \delta t, \Delta \hat{\delta} t]^T$ is transformed to $\mathbf{x} = [\mathbf{r}_{SV}, \dot{\mathbf{r}}_{SV}, \Delta \delta t, \Delta \hat{\delta} t]^T$ through (4). The argument of latitude estimation error covariance $P_u$ is also mapped to the new state-space to obtain the satellite body frame position estimation error covariance $P'_{r_{SV}}$ through

$$P'_{r_{SV}}(t_i|t_i) = \frac{b}{i}R_t(t_i)P_{r_{SV}}(t_i|t_i)\frac{b}{i}R_t(t_i)^T (8)$$

$$P'_{r_{SV}}(t_i|t_i) = \Lambda(t_i)P_u(t_i|t_i)\Lambda(t_i)^T,$$

where $P'_{r_{SV}}$ is the estimation error covariance in ECI; $b_iR$ is the rotation matrix from ECI $\{i\}$ to the SV body frame $\{b\}$; and $b, i, t,$ and $\Omega$ are obtained from SGP4 ephemeris.

Position and Velocity Tracking mode—The closed-loop tracking of the position and velocity states of a LEO SV is performed using an EKF as described in [60]. The estimation state vector is defined as $\mathbf{x} = [\mathbf{r}_{SV}, \dot{\mathbf{r}}_{SV}, \Delta \delta t, \Delta \hat{\delta} t]^T$, and the state update is performed by numerical integration of the two-body with $J_{2}$ model (2). Algorithm 1 details the proposed two-stage LEO satellite tracking framework.

**Algorithm 1 Two-stage LEO satellite tracking**

**Input** $\mathbf{r}_{SV}$, $\dot{\mathbf{r}}_{SV}$, $\rho$

**Output** $\dot{\mathbf{r}}_{SV}$, $\hat{\mathbf{r}}_{SV}$

1. $\dot{\mathbf{r}}_{SV}(t_0) \leftarrow \mathbf{r}_{SV}(t_0)$; $\hat{\mathbf{r}}_{SV}(t_0) \leftarrow \dot{\mathbf{r}}_{SV}(t_0)$
2. Initialize $\dot{\mathbf{u}}(t_0)$ using $\dot{\mathbf{r}}_{SV}(t_0)$ and $\dot{\mathbf{r}}_{SV}(t_0)$ according to (5)
3. $\mathbf{x}(t_0) \leftarrow [\mathbf{u}(t_0), \Delta \delta t(t_0), \Delta \hat{\delta} t(t_0)]$
4. for $t_k = t_0, t_1, ..., t_f$
5. Time update $\dot{\mathbf{u}}(t_k)$ by numerical integration of (6)
6. Measurement update $\dot{x}(t_k)$ and $P(t_k)$
7. end for
8. Initialize $\dot{r}_{SV}(t_1)$ and $\dot{\mathbf{r}}_{SV}(t_1)$ using $\dot{\mathbf{u}}(t_1)$ according to (4)
9. $\hat{x}(t_1) \leftarrow [\dot{\mathbf{r}}_{SV}(t_1), \dot{\mathbf{r}}_{SV}(t_1), \Delta \delta t(t_1), \Delta \hat{\delta} t(t_1)]$
10. Initialize $P'_{r_{SV}}(t_1|t_1)$ according to (8)
11. for $t_k = t_i, t_{i+1}, ..., t_f$
12. Time update $\dot{r}_{SV}(t_k)$ and $\dot{\mathbf{r}}_{SV}(t_k)$ according to (1)
13. Measurement update $\dot{x}(t_k)$ and $P'(t_k)$
14. end for

4. Simulation Results

A simulation study is conducted to evaluate the performance of the proposed tracking framework. Four satellites, pertaining to the Orbcomm, Starlink, OneWeb, and Iridium constellations, were considered.

**Simulation Setup**

The high-fidelity simulator Analytical Graphics Inc. (AGI) Systems Tool Kit (STK) was used to generate satellite orbit trajectories. For each of the four satellites, three different age-of-ephemeris sets were produced: the satellite was propagated forward for around 2, 10, and 20 hours with a time-step of one second to the initial time of visibility to a simulated tracking receiver located in Columbus, Ohio, USA. The LEO SV was propagated using (i) a High Precision Orbit Propagator (HPOP) and (ii) SGP4, yielding two sets of ephemerides for the same satellite, with the HPOP ephemeris serving as the ground truth for the simulation study. The generated LEO satellite trajectories are illustrated in Fig. 2.

![Figure 2. Ground tracks of the simulated LEO satellite trajectories over the tracking receiver. The NORAD ID of each satellite is indicated in brackets.](image-url)
Pseudorange navigation observables to the satellites were generated according to (7). The measurement noise variances were calculated based on the predicted $C/N_0$, which was found from the log distance path loss model

$$C/N_0\rho_i(k) = P_0 - 10 \cdot \log_{10}(d_i(k)/D_0),$$

where $P_0 = 56$ dB-Hz is the nominal $C/N_0$ at a distance $D_0 = 1,000$ km and $d_i(k) = \|r_i(k) - r_{eoc_i}(k)\|$ is the distance between the tracking receiver and the $i$-th LEO SV. The minimum and maximum noise variances for each satellite are proportional to the square root of the inverse $C/N_0$, expressed in linear units, and are listed in Table 1.

Table 1. Minimum and maximum measurement noise variances used in the simulation.

<table>
<thead>
<tr>
<th>NORAD ID</th>
<th>Constellation</th>
<th>$\sigma_n$ [m$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>41186</td>
<td>Orbcomm</td>
<td>0.87 - 4.81</td>
</tr>
<tr>
<td>44761</td>
<td>Starlink</td>
<td>1.27 - 4.03</td>
</tr>
<tr>
<td>45432</td>
<td>OneWeb</td>
<td>1.93 - 4.06</td>
</tr>
<tr>
<td>41926</td>
<td>Iridium</td>
<td>1.03 - 3.55</td>
</tr>
</tbody>
</table>

The filter’s states were initialized using the SGP4-propagated ephemerides at the initial SV visibility time. To demonstrate the benefit of the proposed two-stage tracking framework, the satellite’s dynamic states will be estimated via two methods:

- **Direct Cartesian Tracking**: here, the position and velocity of the SV are tracked in a closed-loop fashion for the entire passing.
- **Proposed Method**: as described in Section 3, the argument of latitude (AOL) of the SV is tracked for the initial 10 seconds of SV visibility time before tracking the position and velocity states for the rest of the passing.

**LEO Satellite Tracking Results**

This subsection presents the tracking simulation results for each LEO satellite for the three age-of-ephemeris scenarios. EKF error plots for selected cases are shown, while Table 4 presents the comprehensive list of results.

Fig. 3 shows the open-loop SGP4 and proposed framework’s EKF position and velocity errors for the Orbcomm LEO SV in the satellite’s body frame. The plots represent the 20 hour age-of-ephemeris case. The initial argument of latitude tracking phase significantly decreases the errors in the along-track position state. The rest of the states are further refined during the position and velocity tracking phase. Moreover, the EKF errors and their associated uncertainty bounds show that the position states in the along-track and radial directions are more observable than those of the cross-track direction. This can be attributed to the LEO SV’s dynamics, where the motion is concentrated in the along-track–radial plane.

Fig. 4 shows the open-loop SGP4 and proposed framework’s EKF position and velocity errors for the Starlink LEO SV, for the 20 hour age-of-ephemeris case. Similarly to the Orbcomm results, the initial argument of latitude tracking phase significantly decreases the errors in the along-track position state. The cross-track position state is refined when compared to the open-loop SGP4 toward the end of tracking duration. The rest of the states are further refined during the position and velocity tracking phase, similarly to the Orbcomm case; but, with slightly decreasing uncertainty bounds in the cross-track position and velocity states, which could be attributed to the orbital geometry relative to the tracking receiver.

Fig. 5 shows the open-loop SGP4 and the proposed framework’s EKF position and velocity errors for the OneWeb LEO SV, for the 20 hour age-of-ephemeris case. The initial argument of latitude tracking phase significantly decreased the errors in the along-track and cross-track position states. The radial position state is further refined during the position and velocity tracking phase. In contrast to the Orbcomm and Starlink results discussed above, the results for the OneWeb SV show a decreasing cross-track uncertainty bound. This behavior could be linked to the fact that OneWeb has a higher orbital inclination ($87.78^\circ$) when compared to Orbcomm and Starlink ($46.86^\circ$ and $53.06^\circ$) (cf. Fig. 2).

Fig. 6 shows the open-loop SGP4 and the proposed framework’s EKF position and velocity errors for the Iridium
LEO SV, for the 20 hour age-of-ephemeris case. The initial argument of latitude tracking phase significantly decreases the high errors in the along-track position and refined the relatively smaller errors in the cross-track position. The error correction during the initial period shows a similar behavior to the OneWeb case which could be due to the similar orbital inclination of Iridium (86.27°). The radial position state is further refined during the position and velocity tracking phase.

Table 4 summarizes the simulation results. The open-loop SGP4 errors gradually increase as the age-of-ephemeris is increased due to longer propagation times resulting in error accumulation. Generally, the error reduction from the proposed framework is more significant when compared to direct Cartesian state tracking for the higher age-of-ephemeris cases.

### 5. Experimental Results

This section presents the results of an experimental study performed with a stationary receiver located in Columbus, Ohio, USA. The receiver opportunistically extracted carrier phase measurements from 4 Starlink, 2 OneWeb, and 1 Orbcomm LEO satellites. Since the accurate or ground truth ephemerides of these satellites was not available, this experiment will demonstrate the practical advantages of the proposed tracking framework by implementing an EKF to localize the receiver using the (i) open-loop SGP4-propagated ephemerides on one hand and (ii) the corrected ephemerides resulting from the tracking on the other hand. Fig. 7 shows the trajectories of the satellites as seen from the receiver.

The initial estimate of the receiver’s position was located at a distance of around 2.8 km away from its true position, with an initial altitude error of less than a meter. In case (i), the SGP4 ephemerides was used, resulting in a final positioning error of about 2.2 km with an inconsistent uncertainty bound. In case (ii), the dynamic states of each LEO satellite were tracked according to the proposed framework using the carrier-phase observables and the produced LEO SV position estimates.
were incorporated into the localization EKF, resulting in a final positioning error of 63.04 m. The localization final errors and their associated 99% uncertainty ellipses are shown in Fig. 8. It is worth noting that in [36], the final positioning horizontal error of 5.1 m was achieved with more SV’s (4 Starlink, 2 OneWeb, 1 Iridium, and 1 OrbcComm). In [36], the SGP4 epoch time was adjusted by minimizing the carrier phase residual for each SV. In contrast to [36], which performed non-causal ephemerides corrections, the achieved results in this paper were achieved with the causal corrections from the proposed tracking framework.

Figure 8. Experimental results showing the final stationary receiver localization estimates and corresponding 99th percentile horizontal uncertainty ellipses using: (i) open-loop SGP4 ephemerides (red) and (ii) the corrected ephemerides (green).

6. CONCLUSION

This paper presented a framework to correct the ephemerides error of non-cooperative LEO satellites. The framework employs pseudorange measurements extracted by a known ground-based receiver to estimate the satellite’s dynamic states in a two-stage process. Simulation results demonstrated the improved LEO tracking performance when compared to standard open-loop SGP4 propagation and direct Cartesian state tracking for multi-constellation LEO satellites and varying age-of-ephemeris scenarios. Experimental results were presented where the tracking framework was implemented on Starlink, OrbcComm, and OneWeb LEO satellites with opportunistically extracted carrier phase observables. Incorporating the tracked ephemerides reduced the receiver’s 3-D localization error from around 2.83 km to 63.04 m.

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