Analysis of Satellite Ephemeris Error in Differential and Non-differential Navigation with LEO Satellites

Joe Saroufim  
Department of Electrical and Computer Engineering  
The Ohio State University  
Columbus, OH 43210  
saroufim.1@osu.edu

Samer Hayek  
Department of Electrical and Computer Engineering  
The Ohio State University  
Columbus, OH 43210  
watchihayek.1@osu.edu

Zaher M. Kassas  
Department of Electrical and Computer Engineering  
The Ohio State University  
Columbus, OH 43210  
kassas.2@osu.edu

Abstract—Low Earth orbit (LEO) satellite ephemeris error is analyzed for standalone (non-differential) and differential navigation. First, the range residual due to ephemeris error is derived for a standalone stationary receiver making range-type measurements to a LEO satellite, leading to deriving upper and lower bounds to this residual. Second, the derived residual is generalized to a differential framework, comprising two receivers making range-type measurements to the same LEO satellite. The differential range residual is found to be minimized whenever the baseline between the receivers is collinear with the projected line-of-sight (LOS) vectors on the local navigation plane, and maximized in the normal direction. Third, the combined effect of the baseline’s orientation and distance is analyzed, where the distance between the two receivers is shown to have no impact on the differential residual along the direction of minimum error. Finally, experimental results are presented to demonstrate the benefit of differential navigation. It is shown that differential navigation significantly reduces the effect of LEO ephemerides errors, achieving a two-dimensional (2D) position root-mean-squared error (RMSE) of 11.7 m, as compared to 54.4 m for the non-differential scenario.

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1. INTRODUCTION

The shortcomings of global navigation satellite systems (GNSS) led researchers over the past decade to study the exploitation of signals of opportunity (SOPs) for positioning, navigation, and timing (PNT) [1]. Meter-level accuracy was achieved with cellular SOPs on ground vehicles [2, 3] and aerial vehicles [4, 5]; while upon employing differential frameworks, sub-meter-level accuracy was shown to be achievable on unmanned aerial vehicles (UAVs) [6, 7].

In recent years, there has been a rapid emergence of space vehicles (SVs) in low Earth orbit (LEO), the majority of which are launched for broadband communications, Internet-of-Things (IoT), and autonomous vehicle connectivity applications [8–12]. Aside from these applications, LEO satellites have received significant interest from the research community, government agencies, and private industry, as key enablers for PNT [13–23]. This is due to their inherently desirable characteristics for PNT. First, the number of LEO satellites is growing dramatically rather quickly, with SpaceX currently leading the race, having around 4,500 active satellites, with a plausible extension to 42,000 [24]. This abundance offers favorable geometric dilution of precision (GDOP), which leads to improved positioning accuracy [25]. Second, LEO SVs have significantly smaller orbiting periods as compared to medium Earth orbit (MEO) satellites, which yields informative LEO Doppler measurements. Third, transmitting from low altitudes results in a higher received signal power than GNSS satellites that reside in MEO.

The recent literature focused on addressing the challenges of exploiting LEO SOPs for PNT. First, LEO signals’ properties are not necessarily disclosed to the public. Nevertheless, several receiver design architectures [26] have been proposed that could produce navigation observables with partially known [27–31] or completely unknown [32–37] LEO signals. Second, LEO SVs are not equipped with highly-stable atomic clocks, are not tightly-synchronized like their GNSS counterparts, and do not transmit their clock errors [38–41]. Third, LEO SVs’ signals are subject to ionospheric and tropospheric attenuation [42, 43]. Finally, unlike GNSS, LEO SVs’ ephemeris are not publicly communicated in their downlink signals. However, an estimate of the LEO SVs’ states can be calculated from the two-line element (TLE) sets, published and updated by the North American Aerospace Defense Command (NORAD). Every TLE set consists of designated and temporal data on the first row, while the second row lists the SV’s standard orbital elements (inclination angle, right ascension of ascending node, eccentricity, argument of perigee, mean anomaly, and mean motion), defined at a single epoch. TLE sets can be used to initialize the simplified general perturbation 4 (SGP4) orbit determination algorithm [44] to estimate the corresponding satellite’s position and velocity at any time epoch. Nevertheless, the estimated Keplerian elements suffer from certain errors, which would accumulate and project into the propagated ephemeris, leading to an error in the satellites’ states, ranging from hundreds-of-meters to a few kilometers. These errors are mostly concentrated in the SV’s along-track direction [45].

The ephemeris error challenge was addressed via the simultaneous tracking and navigation (STAN) approach, where the SVs’ and receiver’s states are simultaneously estimated [46]. Another approach was proposed in [47], which corrected for...
the error by tracking the LEO SVs’ argument of latitude, achieving significant reduction in the ephemeris error.

Another approach to reduce the effect of ephemeris errors on the navigation solution is via differential navigation. Such approach consists of a base and a rover, making measurements to the same satellites. The base is assumed to have a known position and communicates corrections to a rover with unknown states [48]. Differential navigation has been extensively studied for GNSS-based navigation, as a way of significantly reducing common mode errors (e.g., atmospheric delays, clock errors, and ephemeris errors) between the base and the rover. Differential navigation have also been studied for terrestrial SOPs [7, 49]. Recently, differential navigation has shown promising results for LON PNT with carrier phase measurements from Orbcorn LEO [50], Doppler measurements from Iridium [51, 52] and Starlink [53] LEO, and multi-constellation LEO (Starlink, OneWeb, Orbcorn, and Iridium) [54].

Although differential navigation reduces the effect of ephemeris errors, the latter still induce some uncommon error to the measurements at both receivers, which would propagate into the navigation solution. Therefore, modeling LEO SVs’ orbit errors constitutes an essential step in understanding the mapping of these errors into the measurement space. Such models would improve opportunistic differential navigation with LEO satellites. Towards this objective, precise orbit determination and LEO augmentation using GNSS were proposed [55, 56]. The limitation of these methods arises from the need to access an onboard GNSS receiver. Orbit error compensation methods were introduced to improve positioning accuracy using Doppler measurement from Iridium LEO SVs [57]. A study was recently conducted to characterize the effect of the LEO SV along-track error onto the range measurement from a LEO SV, and the extent to which differential positioning can reduce this error propagation to the navigation solution [58]. The study derived an expression for the non-differential and differential range errors, where the analysis was conducted in the satellite’s orbital plane, and validated using simulated pseudorange measurements from Starlink LEO SVs. It was shown that the differential range error varies with the baseline orientation with respect to the receiver–satellite line-of-sight (LOS). However, the study was constrained to an orbital plane analysis and considered two fixed receivers with a constant baseline distance.

This paper builds on [58] and extends the analysis to the local navigation frame to show the actual benefit of differential navigation to compensate for ephemeris errors. This paper makes the following contributions. First, the range residual due to ephemeris error is derived for a standalone stationary receiver making range-type measurements to a LEO satellite, leading to deriving upper and lower bounds to this residual. Second, the derived residual is generalized to a differential framework, comprising two receivers making range-type measurements to the same LEO satellite. The differential range residual is found to be minimized whenever the baseline between the receivers is colinear with the projected line-of-sight (LOS) vectors on the local navigation plane, and maximized in the normal direction. Third, the combined effect of the baseline’s orientation and distance is analyzed, where the distance between the two receivers is shown to have no impact on the differential residual along the direction of minimum error. Finally, experimental results are presented to demonstrate the benefit of differential navigation. A ground vehicle traveled for 486 m in 50 seconds, while aiding its onboard inertial measurement unit (IMU) with differential Doppler measurements from 2 Starlink, 1 Orbcorn, and 1 Iridium LEO satellites, whose erroneous ephemeris were obtained from SGP4, initialized with two-line element (TLE) files. It is shown that differential navigation significantly reduces the effect of LEO ephemerides errors, achieving a two-dimensional (2D) position root-mean squared error (RMSE) of 11.7 m, as compared to 54.4 m for the non-differential scenario.

The rest of the paper is organized as follows. Section 2 presents the non-differential and differential measurement models and maps the LEO SVs’ ephemeris error onto the measurement space. Section 3 presents simulation results of the baseline’s effect on the differential range error. Section 4 shows experimental results of a ground vehicle, demonstrating the performance of differential navigation. Section 5 gives concluding remarks.

2. MAPPING OF LEO SATELLITE’S EPHEMERIS ERROR ONTO THE MEASUREMENT SPACE

This section analyzes the impact of a satellite’s orbit error on range-type measurements (e.g., pseudorange and carrier phase) at a specific epoch. To analyze the mapping of a LEO satellite’s ephemeris error to a pseudorange measurement, perfect clocks and negligible atmospheric attenuation are assumed. Thus, the measurement model is simplified to only the range term, where the propagation error appears. The study focuses on the along-track direction in the satellite’s body frame where most of the error resides. Therefore, for a given orbit, the satellite’s dynamics could be fully defined by the rate of change in the true anomaly, and an ephemeris error projects into a true anomaly error angle $\theta$ shown in Fig. 1. To map this error from the state space to the measurement space, a study is conducted in the LEO satellite’s orbital plane, and shown to hold for all orbits with very small eccentricity where they can be treated as nearly circular. Next, an expression for the orbital plane range error is derived where the point at maximum error is deduced for a full satellite visibility period from a fixed ground receiver. The method is then extended to the differential framework and generalized to the local navigation frame.

![Figure 1. Orbital plane for a non-differential scenario: estimated and actual LEO satellite positions with corresponding range measurements to a stationary receiver projected onto the satellite’s orbital plane.](image-url)
Non-Differential Framework

Let \( \hat{r}_{\text{leo},l} \) and \( \tilde{r}_{\text{leo},l} \) denote the estimated “erroneous” position and velocity of the \( l \text{th} \) LEO satellite obtained from TLE+SGP4 after a relatively long period of propagation; \( r_1 \) and \( \hat{r}_1 \) are the range vectors from the receiver to the \( l \text{th} \) true and estimated LEO SV, respectively. The range error \( \nu \) can be written as

\[
\nu = \| r_1 \| - \| \hat{r}_1 \| = \| r_r - r_{\text{leo},l} \| - \| r_r - \hat{r}_{\text{leo},l} \| .
\]

Orbital Plane Analysis—Define a variable \( \alpha \) as the L2 norm of its corresponding vector form, i.e., \( \alpha = \| a \| \). Also, denote \( r_{\text{op},l} \), \( \hat{r}_{\text{op},l} \) and \( r_{\text{op},r} \) the projections of \( r_1 \), \( \hat{r}_1 \), and \( r_r \), respectively, on the \( l \text{th} \) LEO satellite’s orbital plane. Fig. 1 shows the orbital plane of satellite \( l \) with the projection of the receiver position vector onto this plane. The latter may be defined by its standard elements or the position and velocity vectors of the satellite at any time epoch as

\[
a_{\text{op}} = a - a_{\text{op}_n},
\]

\[
a_{\text{op}_n} = \frac{\langle a, n \rangle}{\| n \|^2} n, \quad n = r_{\text{leo}} \times \hat{r}_{\text{leo}},
\]

where \( a_{\text{op}} \) is the projection of \( a \) on the orbital plane, \( n \) is the relative angular momentum of the satellite, and \( a_{\text{op}_n} \) is the projection of \( a \) along \( n \). Hence, the true orbital plane range can be written as

\[
r_{\text{op}} = \sqrt{\hat{r}_{\text{op}}^2 + e^2 - 2.e.\hat{r}_{\text{op}}.\cos \gamma},
\]

where \( e \) is the error vector of a LEO satellite in its orbital plane defined as

\[
e = \hat{r}_{\text{leo},l} - r_{\text{leo},l} = \hat{r}_l - r_1.
\]

Considering a small eccentricity orbit with an along-track error of a few kilometers, yields \( \hat{r}_{\text{leo}} \approx r_{\text{leo}} \). It can be inferred from Fig. 1 that \( \gamma \) may be written as

\[
\gamma = \varphi + \frac{\pi - \theta}{2}.
\]

Using the above assumptions and a LEO SV position measured from the center of the Earth \( \hat{r}_{\text{leo}} \) to be somewhere in the range 6500-7000 Km, leads to a true anomaly error \( \theta \approx 0.01^\circ \), making the assumption \( \cos \gamma \approx \sin \varphi \) valid. Taking the satellite’s position error to be along the direction of motion, the range error is maximized whenever \( \nu = | r_{\text{op}} - \hat{r}_{\text{op}} | \) is maximized; hence, the only variable over which the maximization takes place is \( \varphi \), leading to

\[
\nu_{\text{max}} = \hat{r}_{\text{op}} \left( \sqrt{1 + \frac{2e \sin \varphi^*}{\hat{r}_{\text{op}}}} - 1 \right)
\]

and

\[
\varphi^* = \begin{cases} 
\arg\max_{\varphi} \sqrt{1 + \frac{2e \sin \varphi}{r_{\text{op}}}}, & \text{for } \delta < 0 \\
\arg\min_{\varphi} \sqrt{1 - \frac{2e \sin \varphi}{r_{\text{op}}}}, & \text{for } \delta > 0,
\end{cases}
\]

where \( \delta \) is the rate of change in the satellite’s elevation angle, which is related to the range rate. To evaluate \( \varphi^* \), consider the 2D polygon formed by \( \hat{r}_{\text{leo}}, \hat{r}_{\text{op}}, \) and \( r_{\text{op},r} \), which yields to

\[
\varphi^* = \begin{cases} 
\max \arcsin \left( \frac{r_{\text{op},r} \times \hat{r}_{\text{leo}}}{r_{\text{op},r} \cdot \hat{r}_{\text{leo}} \sin(\alpha)} \right), \quad \alpha^* = \frac{\pi}{2}, \quad \text{for } \delta < 0 \\
\min \arcsin \left( \frac{r_{\text{op},r} \times \hat{r}_{\text{leo}}}{r_{\text{op},r} \cdot \hat{r}_{\text{leo}} \sin(\alpha)} \right), \quad \alpha^* = -\frac{\pi}{2}, \quad \text{for } \delta > 0.
\end{cases}
\]

The latter suggests that, for a given receiver position, the range error depends on the relative SV position with respect to the receiver, represented by the angle \( \alpha \) in Fig. 1. Hence, the maximum range error occurs whenever the projected receiver position is normal to the estimated range vector in the orbital plane, which is the point of minimum satellite elevation from the receiver. Similarly, the range error is zero whenever \( \varphi \) is zero, corresponding to the maximum satellite elevation angle. To demonstrate this result, range measurements were simulated from two Starlink LEO satellites, with prior knowledge of their true ephemeris, over 300 seconds. Estimated erroneous ephemerides were obtained by perturbing each SV’s ground truth position with an along-track error of 4 km. The range error to each satellite was calculated over the entire visibility period, and shown is Figs. 2(a,b) along with the maximum error bound derived in (1). Figs. 2(c,d) illustrate the elevation of each satellite, showing that the range error is inversely correlated with the elevation angle.

Receiver State Estimation Errors—To assess the propagation of the ephemeris error from range measurement to receiver positioning, a stationary receiver with known position was considered. Pseudorange measurements were simulated from 14 Starlink LEO satellites with prior knowledge of their ground truth ephemeris. Erroneous LEO SVs states were used for the estimated measurements by inducing the same amount of along-track error on their true ephemeris. This discrepancy between the true and estimated measurements will cause the estimator to inaccurately update the receiver states. The analysis may be conducted using the nonlinear least-squares update given by

\[
\Delta x = \left( H^T H \right)^{-1} \Delta z,
\]

where \( \Delta x \) is the state estimate error vector, \( \Delta z \) is the measurement residual vector, and \( H \) is the measurement Jacobian.
matrix given by

$$
H = \begin{bmatrix}
\hat{l}_{r,1} \\
\vdots \\
\hat{l}_{r,L}
\end{bmatrix},
$$

where $\hat{l}_{r,L} \triangleq \frac{r_r - \hat{r}_{r,L}}{||r_r - \hat{r}_{r,L}||}$ is the unit line of sight (LOS) vector from the $l$-th LEO SV.

Fig. 3 shows the 3D position RMSE obtained with hundred different along-track errors induced on every satellite’s states varying from 0 to 4,000 m. It can be seen that the positioning error is at the same order of magnitude as the along-track error, where it reaches 2,900 m with a 4,000 m ephemeris error on all 14 Starlink satellites if not accounted for.

Although a range error can be modeled for a given satellite position, prior knowledge of the along-track error is needed, which is unknown. To overcome this, a differential framework is adopted to reduce the effect of common ephemeris error between two receivers; hence, improving the positioning accuracy.

**Figure 3.** Impact of 14 LEO SVs’ along-track errors on a stationary receiver’s 3D position RMSE.

### Differential Framework

This subsection studies the effect of an along-track error at a single time epoch on the differential measurements from two stationary receivers with known positions, simultaneously extracting range measurements from the same LEO satellite. First, following an analogous approach to equation (1), the range error is calculated at both receivers, showing the relationship between the baseline orientation and the differential range measurement in the satellite’s orbital plane, with a constant baseline distance. Second, this study is generalized and applied to the local navigation frame, namely the East North Up (ENU) frame, where an upper bound on the actual differenced range measurements is defined as a function of the receivers-satellite orientation on the local navigation frame. Third, the algorithm is implemented on a varying baseline distance between the two receivers for four different satellites, showing the combined effect of both the orientation and distance of the baseline on the differential residual.

**Orbital Plane Analysis**—Define $r_{i,r}$ and $b$ as the projection of the $i$-th receiver position and baseline vectors onto the LEO satellite’s orbital plane, respectively, as illustrated in Fig. 4. According to (1), the true range at receiver $i$ on the orbital plane can be calculated as

$$
r_i = \hat{r}_i \left(1 + \frac{2e \sin \varphi_{r_i}}{\hat{r}_i} \right),
$$

and the corresponding differential range residual is then written as

$$
\nu = \hat{r}_1 \left(1 + \frac{2e \sin \varphi_{r_1}}{\hat{r}_1} - 1 \right)
- \hat{r}_2 \left(1 + \frac{2e \sin (\varphi_{r_1} - \varphi_{r_2})}{\hat{r}_2} - 1 \right),
$$

where the first and second terms at the right-hand side of (2) represent the range errors at receiver 1 and receiver 2, respectively, and

$$
\varphi_{r_{1,r}} = \arccos \left(\frac{\hat{r}_1^2 + \hat{r}_2^2 - b^2}{2\hat{r}_1\hat{r}_2}\right).
$$

Equation (3) shows that the angle $\varphi_{r_{1,r}}$, formed by the two estimated LOSs, depends on the estimated ranges and the baseline distance between both receivers. Thus, to study the effect of $\varphi_{r_{1,r}}$ on the range error given in (2), receiver 1 and the baseline distance $b$ are fixed, while receiver 2 can be located anywhere along a distance $b$ from receiver 1. Hence, the differential error becomes solely dependent on $\varphi_{r_{1,r}}$ and the estimated ranges, leading to the relationship between the relative receivers’ orientation and the differential range residual at a specific time epoch. It can be shown from (2) that the residual at receiver 2 eliminates its counterpart at receiver 1 when $\varphi_{r_{1,r}}$ is almost zero, while the differential range error is maximized whenever $\varphi_{r_{1,r}}$ is maximum, hence when $|\psi| \approx \frac{\pi}{2}$, which is the angle between the projected estimated range at the fixed receiver and the projected baseline on the orbital plane. Note that these values of $\varphi_{r_{1,r}}$ that maximize and minimize the differential range error slightly vary for a large baseline distance, when the difference between the estimated ranges increases.

**Figure 4.** Differential scenario in orbital plane: estimated and actual LEO satellite positions with corresponding range measurements to two stationary receivers projected onto the satellite’s orbital plane.

### Topocentric Coordinate System

To make use of this finding, the relation was found to hold in the local navigation coordinate system, known as the
The topocentric coordinate system, where the actual measurements are made. In particular, the same projections explained earlier were applied onto the East-North plane, with the addition of the satellite’s state vector, originally defined in the orbital plane. The topocentric coordinate system is centered at the receiver’s location on the surface of the Earth, determined by its east longitude \( \Lambda \) and geodetic latitude \( \phi \). Since the Earth is a slightly oblate spheroid, the receiver’s position vector from the center of the Earth is not along the normal to the plane, except at the poles and equator. As a result, the normal to the Earth’s surface at the receiver intersects the polar axis slightly below the center of the Earth for any location in the northern hemisphere, and above it for the southern part. The upward axis in the topocentric frame translated along the Z-axis of the ECEF frame can be obtained by

\[
\hat{n} = \begin{bmatrix} R \cos \phi \cos \Lambda \\ R \cos \phi \sin \Lambda \\ R_\phi \sin \phi \end{bmatrix},
\]

where \( R \) is the receiver position vector magnitude in ECEF, and \( R_\phi \) is the vector magnitude normal to the local navigation frame, calculated as

\[
R_\phi = \frac{R_e}{1 - (2f - f^2) \sin^2 \phi},
\]

where \( f \) is the flattening or oblateness of the Earth, related to its eccentricity, and defined as

\[
f = \frac{R_e - R_p}{R_e} \approx 0.00335,
\]

where \( R_e \) and \( R_p \) are the equatorial and polar radii, respectively, which are also the semimajor and semiminor axes of the Earth [59].

Note that a conversion to the Earth Centered Inertial (ECI) frame requires knowledge of the local sidereal time \( \theta_G \) which must be added to the east longitude \( \Lambda \) to account for the Earth’s rotation.

The location of the second receiver with maximum differential range error may hence be specified by taking the cross product of the projected range at the first receiver to the LEO satellite, and the normal \( \hat{n} \) to the plane.

### 3. Simulation Results

This section presents simulation results of two stationary receivers making range measurements to 4 Starlink LEO satellites. First, a constant distance between the two receivers is adopted to study the effect of the baseline direction on the differential range error. Second, the combined effect of the baseline’s distance and orientation on the differential range error is studied to validate the previous findings.

**Effect of Baseline Direction**

Next, a simulation was conducted to demonstrate the relationship between the differential range measurement and the baseline orientation on the local navigation frame. Receiver 1 was fixed at The Ohio State University, Columbus, Ohio, USA, while receiver 2 was simulated to move along 100 different locations centered at receiver 1 with a fixed baseline of 5 km, as shown in Fig. 5(a). Both receivers listened simultaneously to the same Starlink LEO SV at each epoch, producing estimated range measurements. The true satellite’s ephemerides were generated via the Analytical Graphics Inc. (AGI) System Tool Kit (STK) and propagated using High Precision Orbit Propagator (HPOP), while the estimated “erroneous” SV position was simulated with a 4 km along-track error along the direction of motion, where the estimated range measurements were obtained. True and estimated range measurements from both receivers were differenced at a single epoch for every different location of receiver 2. The differential residuals at each baseline orientation are shown in Fig. 5(b) for both the orbital plane and ECEF coordinate system, where the maximum differential errors were recorded when the baseline is normal to the projected LOS on the orbital or East-North planes, respectively. From Fig. 5(b), the maximum differential error was found to be 23 m and 11.5 m in the orbital plane and ECEF coordinate system, respectively. However, negligible differential residuals were recorded whenever the baseline is colinear with the projected estimated LOS at the fixed receiver.

**Combined Effect of Baseline Distance and Direction**

Besides the orientation of the receivers with respect to the LEO SV, the impact of the baseline distance on the differential range error is studied. With the same along-track error of 4 km, receiver 2 was then simulated to move in the East-North plane in a 20 km interval from each direction, centered at receiver 1. The differential range error is calculated at each of these positions for 4 different Starlink satellites, and the

![Figure 5](image-url)
results are shown in Fig. 6(b). For each of the 4 SVs in Fig. 6(a), the differential range error is found to be minimized whenever the second receiver is located anywhere along the projected LOS at the first receiver onto the local navigation frame, irrespective of the baseline distance, and maximized along the normal direction, which agrees with the previous finding. Also, along the direction of maximum error, the latter grows with the increase in baseline distance, at a different rate for every satellite. The directions of minimum error, represented by the dark blue stripes in Fig. 6(b), correspond to the LOS direction to each satellite. Hence, the inclination of these stripes indicates the location of each LEO SV along its orbit shown in Fig. 6(a) at the time of the study. According to Fig. 6(b), the maximum error for the 4 SVs ranges from 14 m with SV2 and SV4, to 80 m with SV3 at the largest baseline distance and direction of maximum error. Therefore, with the same along-track error of 4 km, the measurement residual is reduced from around 4,000 m using the non-differential framework to a maximum of 80 m using the differential framework, with a baseline of around 14 km.

Fig. 7 shows the base station’s location, the LEO SVs’ trajectories, and the ground vehicle’s true and estimated trajectories obtained using differential LEO-aided INS, non-differential LEO-aided INS, and GNSS-INS solutions. The differential framework achieved 11.74 m in 2D position RMSE, as compared to 54.39 m for the non-differential framework, and 74.60 m for GNSS-INS after GNSS cutoff. These results show the significant reduction in measurement errors by using a differential framework, leading to an improvement in the navigation solution. The vehicle trajectory and results are summarized in Tables 1 and 2, respectively.

4. EXPERIMENTAL RESULTS

This section demonstrates the benefit of the differential framework, as a promising method to reduce the effect of ephemeris errors and improve the navigation solution. The experiment considered a fixed base station with prior knowledge of its position at the Electroscience Laboratory, at The Ohio State University, Columbus, Ohio, USA, and a ground vehicle navigating with differential Doppler measurements from 2 Starlink, 1 Orbcomm, and 1 Iridium LEO SVs. Note that since pseudorange measurements were not available from these LEO satellites, Doppler measurements were adopted instead. The vehicle traversed a distance of 486 m in 50 seconds in Columbus, Ohio, and its ground truth was obtained from a Septentrio AsteRx SBi3 Pro+ integrated GNSS-INS (inertial navigation system) system with real-time kinematic (RTK) corrections, and an industrial-grade inertial measurement unit (IMU). The base and rover were within a 1.5 km baseline over the whole duration; hence, atmospheric attenuation was negligible. GNSS signals were available for the first 7 seconds, then made unavailable for the remaining 43 seconds, during which the ground vehicle traveled a distance of 376 m. An estimate of the SVs’ ephemerides was obtained from TLE+SGP4 propagation at the time of the experiment.

An extended Kalman filter (EKF) is implemented to estimate the receiver’s states as well as the relative clock bias and drift errors between the rover and base station. The EKF formulation, IMU model, clocks model, as well as the differential and non-differential Doppler measurement models adopted are found in [60].

Fig. 7 shows the base station’s location, the LEO SVs’ trajectories, and the ground vehicle’s true and estimated trajectories obtained using differential LEO-aided INS, non-differential LEO-aided INS, and GNSS-INS solutions. The differential framework achieved 11.74 m in 2D position RMSE, as compared to 54.39 m for the non-differential framework, and 74.60 m for GNSS-INS after GNSS cutoff. These results show the significant reduction in measurement errors by using a differential framework, leading to an improvement in the navigation solution. The vehicle trajectory and results are summarized in Tables 1 and 2, respectively.
Table 1. Vehicle trajectory

<table>
<thead>
<tr>
<th>Metric</th>
<th>Total</th>
<th>No GNSS</th>
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<tbody>
<tr>
<td>Distance [km]</td>
<td>0.486</td>
<td>0.376</td>
</tr>
<tr>
<td>Time [s]</td>
<td>50</td>
<td>43</td>
</tr>
</tbody>
</table>

Table 2. Experimental results

<table>
<thead>
<tr>
<th>Framework</th>
<th>RMSE [m]</th>
<th>Final error [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>GNSS-INS</td>
<td>74.60</td>
<td>209.19</td>
</tr>
<tr>
<td>Non-diff. LEO-aided INS</td>
<td>54.39</td>
<td>148.52</td>
</tr>
<tr>
<td>Diff. LEO-aided INS</td>
<td>11.74</td>
<td>12.01</td>
</tr>
</tbody>
</table>

5. CONCLUSION

This paper studied the impact of an orbital error, concentrated in the along-track direction of a LEO SV’s body frame, on the range measurement extracted by a stationary receiver. Upper and lower bounds on the range error were derived, and this error was mapped to the receiver positioning error. The analysis was conducted for a standalone (non-differential) case in the satellite’s orbital plane, then extended to the topocentric coordinate system for the differential case, showing the reduction in measurement error and the effect of the baseline vector orientation on the differential residual. It was shown that the differential range error is minimized whenever the baseline is colinear with the projected LOS vectors on the local navigation frame, regardless of the baseline distance. Finally, experimental results were presented to demonstrate the efficacy of differential navigation. The experiment considered a ground vehicle navigating for 486 m in 50 seconds, while aiding its IMU with differential Doppler measurements from 2 Starlink, 1 Orbcomm, and 1 Iridium LEO SVs, achieving a 2D position RMSE of 11.7 m, as compared to 54.4 m for the non-differential scenario.

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REFERENCES


**Biography**

**Joe Saroufim** is a Ph.D student in the Department of Electrical and Computer Engineering at The Ohio State University and a member of the Autonomous Systems Perception, Intelligence, and Navigation (ASPIN) Laboratory. He received a B.E. in Mechanical Engineering from the Lebanese American University. His current research interests include low Earth orbit satellites, situational awareness, autonomous vehicles, and sensor fusion.

**Samer Hayek** is a Ph.D student in the Department of Electrical and Computer Engineering at The Ohio State University and a member of the ASPIN Laboratory. He received a B.E. in Mechanical Engineering from the Lebanese American University. His current research interests include low Earth orbit satellites, autonomous vehicles, sensor fusion, and simultaneous localization and mapping.

**Zaheer M. Kassas** is the TRC Endowed Chair in Intelligent Transportation Systems, Professor of Electrical & Computer Engineering at The Ohio State University, and Director of the ASPIN Laboratory. He is also director of the U.S. Department of Transportation Center: CARMEN (Center for Automated Vehicle Research with Multimodal AssurEd Navigation), focusing on navigation resiliency and security of highly automated transportation systems. He received a B.E. in Electrical Engineering from the Lebanese American University; an M.S. in Electrical and Computer Engineering from The Ohio State University, and an M.S.E. in Aerospace Engineering and a Ph.D. in Electrical and Computer Engineering from the U.S. Department of Transportation Center in Texas at Austin. He is a recipient of the National Science Foundation (NSF) CAREER award, Office of Naval Research (ONR) Young Investigator Program (YIP) award, Air Force Office of Scientific Research (AFOSR) YIP award, IEEE Walter Fried Award, Institute of Navigation (ION) Samuel Burk Award, and ION Col. Thomas Thurlow Award. He is Fellow of the IEEE, a Fellow of the ION, and a Distinguished Lecturer of the IEEE Aerospace and Electronic Systems Society. His research interests include cyber-physical systems, navigation systems, autonomous vehicles, and intelligent transportation systems.