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Modeling and Analysis of Group Dynamics in Alcohol-Consumption Environments

Luis Felipe Giraldo, Kevin M. Passino, *Fellow, IEEE*, and John D. Clapp

Abstract—High-risk drinking is considered a major concern in public health, being the third leading preventable cause of death in the United States. Several studies have been conducted to understand the etiology of high-risk drinking and to design prevention strategies to reduce unhealthy alcohol-consumption and related problems, but there are still major gaps in identifying and investigating the key components that affect the consumption patterns during the drinking event. There is a need to develop tools for the design of methodologies to not only identify such dangerous patterns but also to determine how their dynamics impact the event. In this paper, based on current empirical evidence and observations of drinking events, we model a human group that is in an alcohol-consumption scenario as a dynamical system whose behavior is driven by the interplay between the environment, the network of interactions between the individuals, and their personal motivations and characteristics. We show how this mathematical model complements empirical research in this area by allowing us to analyze, simulate, and predict the drinking group behaviors, to improve the methodologies for field data collection, and to design interventions. Through simulations and Lyapunov stability theory, we provide a computational and mathematical analysis of the impact of the model parameters on the predicted dynamics of the drinking group at the drinking event level. Also, we show how the dynamical model can be informed using data collected *in situ* and to generate information that can complement the analysis.

Index Terms—Consensus, drinking groups, Lyapunov stability theory, networks, public health, social dynamics.

I. Introduction

Heavy alcohol consumption is the cause of approximately 1800 deaths per year among college students and is considered a major public health issue in the United States [1]. During the last 20 years, researchers have tried to understand the etiology of heavy drinking among this population and design strategies to intervene to reduce heavy drinking and its consequent problems [2]. The main goal is to find the "leverage points" of the heavy drinking event, that is, the places and times in the event that are crucial for intervention.

Several studies have been conducted to examine the behavior of groups that are in naturally occurring drinking events *in situ* [3]–[5]. These studies have found that the dynamics of alcohol consumption are affected by the characteristics of the individual (e.g., drinking motivations and history), event-level factors (e.g., duration of the drinking event or playing drinking games), and environmental factors (e.g., dancing, food, or drink specials). Statistical tools have been employed to analyze how these factors are correlated and to determine their significance on drinking behaviors [6]. Even though these studies are very important for better understanding high-risk drinking, they are very expensive, difficult to conduct, and they have major gaps. For example, the impact of the interaction between the group network and the individual motivations in the dynamic environment has not been measured or modeled. The social interactions play an important role during the drinking activity [6]–[8], and studying them is critical to understanding the etiology of high-risk alcohol consumption. There is a need to develop methodologies to not only identify the most important factors that affect the alcoholconsumption patterns, but also to determine how they influence the dynamics of the group throughout the drinking event. This is done here.

This need has led to an increasing interest in developing dynamical system models as tools to complement empirical research that addresses not only alcohol-related problems but also public health issues in general [9], [10]. Models of dynamical systems provide a way to analyze comprehensively the problem settings, to develop more effective intervention designs and evaluation methods, and to plan large-scale field studies. As part of the research in public health that works to develop strategies for the effective design of interventions that reduce high-risk alcohol consumption, dynamical models have been proposed to characterize how drinking patterns are affected by the social interactions in large groups [11], [12]. These models describe how the alcohol usage in large populations changes between categories such as "heavy-drinkers," "social-drinkers," and "nondrinkers." The formulation of these models is closely related to the ones employed to describe the dynamics of infectious diseases that spread in a population. Also, several simulation methodologies have been designed to recreate drinking scenarios [13], [14].

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The authors are with the Department of Electrical and Computer Engineering, and the College of Social Work, Ohio State University, Columbus, OH 43210 USA (e-mail: lfgiraldot@gmail.com; passino.1@osu.edu; clapp.5@osu.edu).

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Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

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However, to our knowledge, building a model that describes how the blood alcohol content (BAC) level in a group changes over time during the drinking event and permits analysis not only through simulations but also at the mathematical level for a comprehensive understanding of the modeled behaviors has not been studied yet.

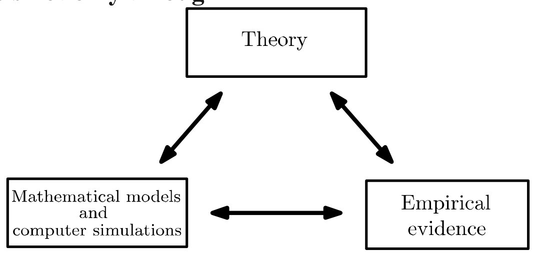


Fig. 1. Conceptual diagram describing the interplay between theory, empirical evidence, and mathematical and computational models in the study of the dynamics of social systems. In that spirit, this paper uses previously collected field data on drinking, theoretical assumptions of group behavior, and a model of a dynamical system along with computer simulations to advance our understanding of the etiological ecology of drinking events.

Here, we construct, based on current observations and empirical data on drinking groups, a model of a system that characterizes how the dynamics of the social interactions, individual characteristics, and environment translate into changes in the drinking patterns of individuals measured through the BAC level. We derive a formulation of the model and a mathematical analysis of the behaviors that can be characterized, and show how this model could complement empirical research by informing theory and testing constructs. In Fig. 1, we present a conceptual diagram of how the theoretical analysis of drinking groups interacts with the empirical research and mathematical and computational system models.

A. Modeling Group Dynamics

In the theoretical analysis of groups in social psychology, the behavior of the group is assumed to be influenced by the environment and the mutual interactions between the group members. This is described by Lewin, who is considered the father of social psychology, through the formula $B = f(P,H)$, where the individual's behavior B is a function f of the personal characteristics and preferences P, and external influences that include the environment and other people H [15], [16, p. 17]. Our previous field studies on drinking events are consistent with this description of group dynamics. These studies have shown that the personal preferences on drinking is not the only mechanism that drives the behavior of an individual in a drinking group. For example, using a portal design [17], we studied the group drinking behavior in bars [3]. It was observed that college students usually drink in settings that vary in risk and protective factors from heavy drinking [6], [18]. Patrons were interviewed and breath alcohol samples were taken upon entering and exiting the bar. When bar patrons are asked their intended level of intoxication when entering a bar, their level of intoxication often failed to match their previously stated intentions (e.g., to get very drunk) once they exited the bar. Fig. 2 shows the measured BAC using a breathalyzer when entering and exiting the bar for four different categories of the intended intoxication level. Although the average BAC does tend to change across categories, it is clear that the BAC that is reported when exiting the bar does not always match the intended level of intoxication. These observations suggest that the behavior of individuals in a group during the drinking event is affected by additional basic mechanisms. We want to propose then a model that describes the dynamics of groups in drinking environments that is consistent with these observations and the theoretical analysis of group dynamics.

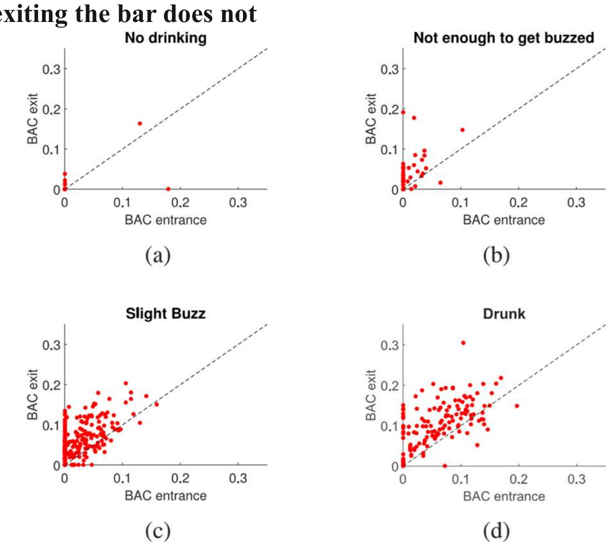


Fig. 2. Measured BAC on subjects before entering the bar (horizontal axis) and after they leave it (vertical axis). Each dot corresponds to the BAC sample obtained from an individual, and the dashed line corresponds to a 45◦ line for reference. The desired level of alcohol that the subjects reported before entering the bar is categorized as (a) not drinking, (b) not drinking enough to get buzzed, (c) slight buzz, and (d) drunk.

Current studies where data are collected *in situ* during the drinking events, although they are very informative, they fall short of describing the drinking patterns throughout the drinking event. For example, in the study reported in [3], the BAC level is measured only before and after the drinking event, but there is no information of how the BAC levels change in between. However, current technology has now made accessible new types of information that helps us overcome several of these limitations. It allows us to have a real-time monitoring of the BAC level [19], social interactions [20], [21], and location in space [22]. Our medium-term goal then is to design and conduct new studies where observational and self-report survey data are collected along with measurements that quantify the dynamics of the drinking group. The contribution of this paper is to provide a mathematical model that, based on up-to-date observations and empirical data of drinking groups, we hypothesize captures the dynamics of the BAC level given the mechanisms that drive the behavior of the group, and can be updated once we collect data for this purpose. In the same way conventional physics tries to explain how force translates into changes in motion, our aim is to create a model that explains the "physics" of the drinking event: how the influences from the individual's personal preferences (e.g., desired effect of the BAC level on his/her body), other members of the group, and the environment translate into changes in the BAC level. We do not claim that this is a perfect model. This paper is just a step in the cyclic process depicted in Fig. 1: given current empirical evidence and theory of group dynamics, we propose a model that improves our understanding of the drinking event and allows us to design better methodologies for the collection of new evidence. This evidence will eventually be used to evaluate the model and make the corresponding improvements/changes on the hypotheses concerning the mechanisms that drive the event dynamics and the construction of intervention strategies.

The proposed model follows a well-developed mathematical framework used in engineering to study the dynamics of multiple interactive agents and their stability properties [23]–[25]. Under this framework, we are able to incorporate in a dynamical system the relationship between the BAC level and the individuals' personal characteristics, the environment they are in, and the influence network between the members of the drinking group (Section II). Using computer simulations and Lyapunov stability theory [26], we present a computational and mathematical analysis of how the parameters affect the modeled dynamics (Sections III and IV), and a discussion of the importance of these results in the further design of interventions and methodologies for the prevention of high-risk alcohol consumption (Section V).

II. Construction of the Model

Our goal is to model how an individual regulates his/her own BAC given the current state of the BAC and the influence of his/her personal motivations, other individuals, and the environment the individual is in. We explain step by step how the model is constructed, starting from the simpler case where the individual is only influenced by his/her own motivations, and extend it to the case where the group and the environment affect the dynamics of the individuals.

A. Individual Influences on Behavior

For the construction of the model, we quantify the behavior of an individual using his/her BAC level and its rate of change varying over time. Let $x_i(t) \geq 0$ denote the BAC level of individual i at time $t \geq 0$, and let $v_i(t) \in \mathbb{R}$ denote its rate of change. We start by assuming that there is no environmental and group pressures, and that individual i has a unique desired BAC level x_i^*. Research suggests that individual factors such as drinking motives (e.g., desired outcome and alcohol level) and drinking history (frequency of heavy drinking) influence the level of intoxication in the individual at the event-level [18], [27]. We assume therefore that x_i^* is chosen by the individual in accordance with his/her personal motives and characteristics. The model then should describe the behavior of an individual such that he/she tends to regulate his/her own BAC to reach the desired level. This means that if the actual BAC is below x_i^*, then the dynamics of the individual (e.g., amount of alcohol consumed) should be such that there is an increase in his/her BAC level. On the other hand, if the actual BAC is above x_i^*, then the dynamics of the individual should be such that there is a decrease his/her BAC level. An initial model that captures such dynamics is shown in the causal loop diagram in Fig. 3. In this model, it is assumed that the individual regulates his/her change rate of

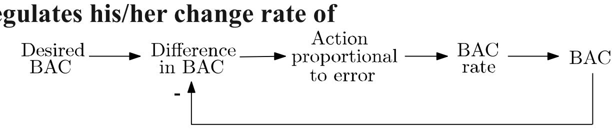


Fig. 3. Causal loop diagram of the dynamics of the BAC modeled using a first-order system. The action proportional to error corresponds to the product of the difference in BAC and a scaling factor that determines how quickly the individual reaches the desired BAC level. In this model, it is assumed that there are no group and environmental pressures. The causal links that have no sign are assumed to have positive polarity.

the BAC proportional to the difference between the desired level and actual level. The constant equilibrium point of the dynamical system is x_i^*. The differential equation associated with the diagram in Fig. 3 is given by $\dot{x}_i(t) = \eta_i^p (x_i^* - x(t))$, where $\dot{x}_i$ is the derivative of x_i with respect to t, $v_i(t) = \dot{x}_i(t)$, and $\eta_i^p > 0$ is the proportional action parameter, which determines how quickly the individual changes his/her BAC, and can be seen as the commitment of individual i to reach the desired level.

Even though this model captures the basic behavior of an individual that wants to reach his/her desired BAC level, it is restrictive with respect to the dynamics that can be represented. An example is the fact that the BAC trajectories modeled using the system in Fig. 3 will never exhibit an overshoot with respect to the desired BAC level, a situation that is actually possible in drinkers who are not able to regulate accurately the alcohol consumed during the drinking event. A model formulation that generates a richer set of trajectories of the BAC level, including the possibility of modeling overshoot, is presented in Fig. 4, which is an extension of the one in Fig. 3 where now the dynamics of the individual are such that the difference between the desired level and the actual BAC affects the acceleration of the BAC, that is, it directly affects how $\dot{v}_i$ changes over time. According to the diagram in Fig. 4, an individual tends to accelerate his/her BAC when the actual BAC is below the reference, and decelerate when it is above. Also, an individual is assumed to restrain the BAC acceleration depending on his/her perception on how quickly the BAC level is changing. This last component of the model is the one that shapes the trajectory and allows the behavior of the BAC level to have overshoot or not. The set of differential equations that represents this dynamical system is given by

$$\dot{x}_i(t) = v_i(t)$$
$$\dot{v}_i(t) = \eta_i\left(x_i^* - x_i(t)\right) - \zeta_i v_i(t) \quad (1)$$

where $\zeta_i > 0$. Note that an individual accelerates or decelerates his/her BAC depending on the current value of $v_i(t)$ and where the actual BAC level is with respect to the desired one. Parameter η_i determines how strong the commitment of the individual to reach the desired BAC level is [i.e., commitment to make $x_i^* - x_i(t) = 0$], and parameter ζ_i can be seen as the strength of the opposition of individual i to quick variations of the BAC level. Fig. S1 in the supplementary file shows instances of BAC trajectories that can be generated for different values of η_i and ζ_i, given x_i^*.

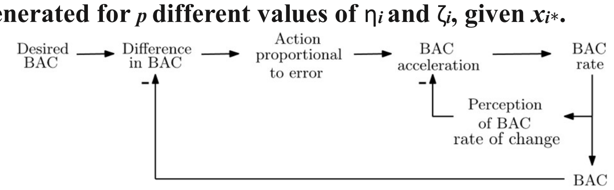


Fig. 4. Causal loop diagram of second-order system that models the dynamics of the BAC level. It is assumed that there are no group and environmental pressures.

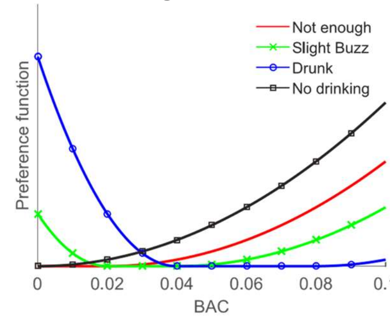
Although the dynamical system in Fig. 4 with differential equation in (1) allows for modeling a wide variety of behaviors, the assumption that the individual has a unique desired BAC level is restrictive. It can be the case that the objective of an individual is to reach any BAC level within an interval that produces certain body reactions. For example, a person who wants to have a "slight buzz" effect will typically reach BAC levels that are between 0.02 and 0.04 [28, Ch. 4]. To be able to generalize our formulation to include these cases in the model, we introduce the personal preference function.

Let $f_i^p : \mathbb{R} \rightarrow \mathbb{R}$ be a continuously differentiable function that quantifies how preferable a given BAC is for individual i. We assume that f_i^p has a unique minimum, and without loss of generality, $\min_x f_i^p(x) = 0$. Lower values of $f_i^p(x_i)$ indicate more preferable BAC levels for the individual. For example, if individual i wants to get drunk, then higher values of x_i will correspond to lower values of $f_i^p(x_i)$. The influence of an individual's personal preference on his/her behavior is then given by the negative derivative of function $f_i^p(x)$ with respect to x, since it points to the direction where $f_i^p(x)$ decreases. This means that $-df_i^p(x_i(t))/dx_i$ indicates whether the individual should increase or decrease his/her BAC in order to reach the desired level. It is implicit in the system in Fig. 4 and (1) that individual i has a function $f_i^p(x_i(t)) = (1/2)(x_i(t) - x_i^*)^2$, where (1) can be rewritten as

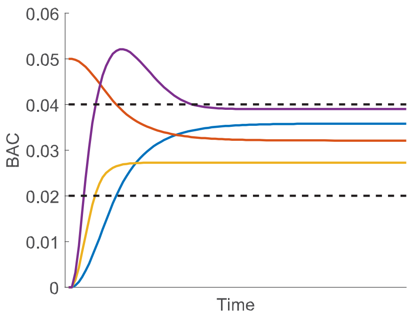
$$\dot{x}_i(t) = v_i(t)$$
$$\dot{v}_i(t) = -\eta_i \frac{df_i^p}{dx_i}(x_i(t)) - \zeta_i v_i(t). \quad (2)$$

In the specific case of the system in (1), the negative derivative of the preference function with respect to x_i is given by $-df_i^p(x_i(t))/dx_i = x_i^* - x_i(t)$, which corresponds to the difference in BAC part of the model. Note that in this case $f_i^p(x_i(t))$ has a unique minimum at x_i^* (the most desirable BAC level, where the derivative is zero) and is symmetric around this point. Instead of modeling the influence of the individual on his/her own behavior to reach the desired BAC level through the term $\eta_i(x_i^* - x_i(t))$, we model it in a more general way using the negative derivative of the preference function, where now the goal is to reach a BAC level such that $-df_i^p(x_i(t))/dx_i$ is zero.

Using the concept of preference function, we can take empirical evidence that categorizes the effect of BAC level on an individual's body to construct different function profiles that describe how an individual tends to regulate his/her BAC to reach the desired effects. Fig. 5(a) shows the preference profiles for four categories of alcohol intoxication: "not enough to get buzzed," slight buzz, "drunk," and "no drinking."



(a)



(b)

Fig. 5. (a) Preference functions associated with the categories of the body's reaction to the BAC level: not enough to get buzzed, slight buzz, drunk, and no drinking. The formulation of these functions is presented in Table S1 in the supplementary file. (b) Trajectories of the BAC level for different initial p conditions and values of η_i and ζ_i. In this case, the individual's desired effect is slight buzz, which implies that the final BAC level is within the interval [0.02,0.04], which is where the derivative of the corresponding preference function in (a) is zero. The MATLAB source code to generate these plots is available in Section E of the supplementary file.

The mathematical formulation of these functions is presented in Table S1 in the supplementary file. Note that the minimum values of these profiles correspond to the intervals of the BAC level that have been reported to produce the respective effects. BAC levels in the interval [0,0.02] are associated with category not enough to get buzzed, interval [0.02,0.04] with slight buzz, values greater than 0.04 with drunk, and 0 with no drinking [28, Ch. 4]. The slope of the functions is chosen to be larger for values of the BAC that are before the interval where the function is minimum than those that are after the interval, implying that the individuals are more committed to increase their BAC to reach the desired effect than to decrease it. Fig. 5(b) shows example trajectories modeled using (2) of individuals whose preferred effect of the BAC level is slight buzz. All the trajectories converge to the interval of the BAC level [0.02,0.04], instead of converging to a single BAC level, due to the appropriate choice of the preference function.

The dynamical system in (2) enables us to model a variety of trajectories that the BAC level of an individual can have in a drinking event. It has the flexibility to model the commitment of the individual to approach the desired levels, his/her own perception of the change rate to restrain quick changes on his BAC, and also through the personal preference function we can define intervals of desired levels that are associated with effects on the individual's body. The dynamical system in (2) can be seen from a physics perspective. As in Newton's second law, there are "forces" acting on the individual's acceleration. The concept of forces is not necessarily the one measured in Newtons, but it refers to influences that cause a change in the individual's BAC level, which in this case correspond to the influence of the individual to reach the desired BAC levels and the influence that restrains the individual to quick changes in his/her BAC according to his/her perception of the BAC rate of change. We will prove in Section IV-A that this last component (perception of the BAC rate of change) is necessary in the model for the convergence of the individual's BAC level to the desired interval.

B. Adding Social and Environmental Influences

In the study of human group dynamics, it has been observed that the behavior of an individual is determined not only by his/her own personal characteristics but also by the influences of the group and environment he/she is in [15] and [16, p. 17]. This has been already observed in the particular case where the group is in an alcohol-consumption environment. It has been shown that the social interactions influence the dynamics of the individuals, where the patterns of communication between group members and the strength of the influence of other members on an individual play an important role in shaping the behavior of the whole group [29], [30]. Also, data collected *in situ* provide evidence that environmental factors have a significant effect on the dynamics of the drinking groups [31], [32]. For example, an environment where there are large crowds has an impact on the behavior such that there is a tendency to restrain increases in the BAC. On the other hand, environments with drinking games promote higher BAC levels.

In the same way the personal preference function is used to include in the model the influence of the individual on his/her own behavior to reach the desired BAC levels, we define functions to describe how the environment and the group affect the behavior of individual i. First, to model the influence of the group on an individual's behavior, we consider evidence that suggests that in a group of people there are mutual attractions that lead the group toward consensus in drinking behavior [29], [33]. This means that, in the context of our model, an individual is attracted to the BAC level and its change rate of other individuals. Since an individual might interact with only a subset of the group and the impact of those interactions can vary in strength between people, we need to formally define an influence network in the drinking group. Assume that the drinking group has n members. The structure of the network is given by $\mathrm{G} = (\mathrm{V},\mathrm{E},\mathrm{W})$, where $\mathrm{V} = \{1,\dots,n\}$ is the set of labels for each individual in the group, and $\mathrm{E} \subset \mathrm{V} \times \mathrm{V}$ is the set of directed links that connect the individuals. Link $(i,j) \in \mathrm{E}$ indicates that individual j influences individual i. It might be the case where $(j,i) \in \mathrm{E}$ does not exist, meaning that there is no influence from i to j. Let $N_i = \{j \in \mathrm{V} : (i,j) \in \mathrm{E}\}$ be the set of all the group members that have some influence on individual i.

Each link (i,j) is associated with a weight $w_{ij} > 0$ that corresponds to the strength of the influence of j on i. We have that $\mathrm{W} = \{w_{ij} : i \in \mathrm{V}, j \in N_i\}$ is the set of all the weights associated with the links in E. The mathematical formulation of the social interactions in this context is in terms of attractions. We say that individual i is influenced by j if there is a tendency from i to follow the BAC level and/or its rate of change of individual j. It is assumed that each individual is able to estimate the BAC level and its rate of change of the group members that influence him/her (i.e., those that are in the set N_i). The function associated with the group influence is formulated as

$$f_i^g(x_i(t),v_i(t)) = \frac{1}{2}\sum_{j\in N_i} w_{ij}\left[\left(x_i(t)-x_j(t)\right)^2 + b\left(v_i(t)-v_j(t)\right)^2\right]$$

where $b \geq 0$ is a parameter associated with the influence on the rate of change of the individual's BAC level. This function is minimized when individual i tends to follow his/her neighbors' BAC level and its rate of change. Larger values for w_{ij} imply a larger attraction of individual i to follow j. The influence component associated with the social pressures will be then a_i^g given by the negative derivative of f_i with respect to both x_i and v_i

$$a_i^g(t) = -\frac{df_i^g}{dx_i}(x_i(t)) - \frac{df_i^g}{dv_i}(v_i(t)) = -\sum_{j\in N_i} w_{ij}\left[\left(x_i(t)-x_j(t)\right) + b\left(v_i(t)-v_j(t)\right)\right]. \quad (3)$$

Term $-w_{ij}[(x_i(t)-x_j(t))+b(v_i(t)-v_j(t))]$ is a value that points to the direction where $x_j(t)$ and $v_j(t)$ are with respect to $x_i(t)$ and $v_i(t)$ and describes the influence of j on the behavior of i. This part of the model is the one that allows us to characterize the drinking group as an interconnection of dynamical systems, that is, a dynamical social system.

To define the influence of the environment on the group's drinking behavior, we define a continuously differentiable function $f^e : \mathbb{R} \rightarrow \mathbb{R}$ that is associated with the environment the group is in. The environment promotes BAC levels where the function f^e takes lower values. For example, an environment where there are large crowds will have a function profile with lower values at lower BAC levels. The influence of the environment on individual i is given by the scaled negative derivative of this function

$$h_i(t) = -\eta_i^e \frac{df^e}{dx_i}(x_i(t)) \quad (4)$$

where $\eta_i^e \geq 0$ is the strength of the influence of the environment on individual i. Equation (4) characterizes the environmental pressures that affect the behavior of the drinking group.

Using the results in (3) and (4) along with (2), we construct a model that characterizes how influences of the group, environment, and individual preferences on the BAC effects translate into changes in a person's BAC level. To complete the model, we assume that the action of these three components

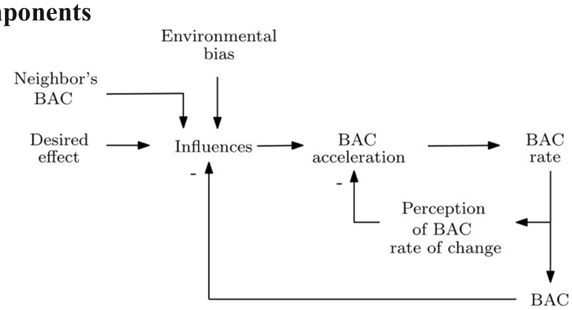


Fig. 6. Causal loop diagram of the final model that accounts for the effect of the individual characteristics, group, and environment on changes in the BAC level.

on behavior is additive. The complete model is illustrated in Fig. 6 and defined in the set of equations

$$\dot{x}_i(t) = v_i(t)$$
$$\dot{v}_i(t) = p_i(t) + a_i(t) + h_i(t) + d_i(t) \quad (5)$$

where $p_i(t) = -\eta_i^p \frac{df_i^p}{dx_i}(x_i(t))$, and $d_i(t) = -\zeta_i v_i(t)$.

III. Simulation Results

The mathematical formulation of the group dynamics in (5) allows us to characterize behaviors that have been observed in human groups involved in alcohol-consumption activities. Next, through simulations, we show the dynamics that can result for different choices of the desired effect of the alcohol intoxication and the structure of the influence network in the drinking group. Also, we study data from real drinking groups that were collected *in situ*. We use this information to inform our model and make predictions about the behavior of the group given some assumptions on the parameters of the model.

A. Group Dynamics Under Different Conditions on the Personal Preferences and Influence Networks

In the first set of simulations, we study the dynamics of a group of six people in three different scenarios. Individuals 1–3 have slight buzz as the preferred effect of the BAC level, meaning that these individuals prefer values of x within the interval [0.02,0.04]. On the other hand, "Not drinking" is the preference for individuals 4–6, which means that they prefer having values of x that tend to zero. The profiles of the preference functions for these two subgroups are shown in Fig. 5(a). The implementation details and source code of the simulations that we present in this section are given in Sections C and F of the supplementary file.

In the first case, the group has a network structure as shown in Fig. 7(a). Each arrow indicates the direction of the influence between a pair of individuals, and its thickness is proportional to the strength of the influence [i.e., w_{ij} in (3)]. The only interaction between subgroups occurs between 3 and 4. The influence of 3 on 4 is stronger than the influence in any other pair of individuals. There is no influence of 4 on 3. Fig. 7(b) shows the dynamics of the group given the initial conditions and model parameters. Note that the individuals whose preference is no drinking initially tend to have lower values of the BAC level even though some of them start at high values.

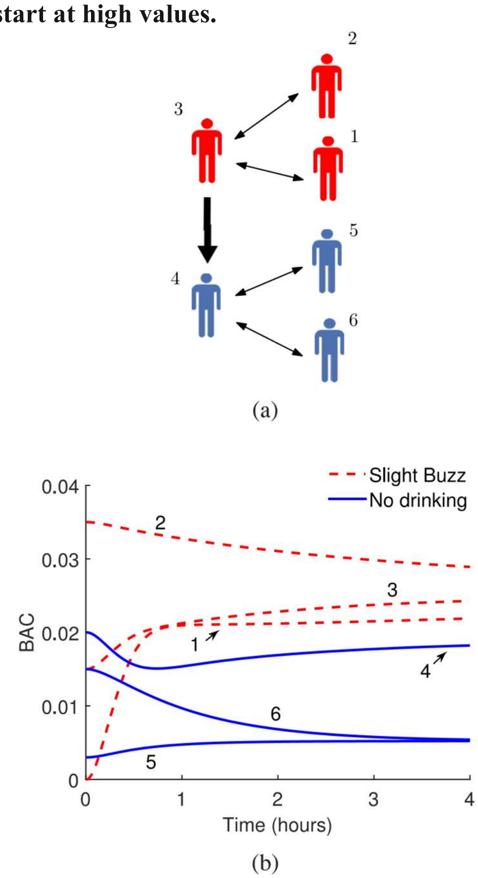


Fig. 7. (a) Influence network in a group where individuals 1–3 have slight buzz as desired effect of the BAC level while individuals 4–6 prefer being in the category no drinking. (b) Trajectories of the simulated BAC level for each one of the individuals in the group.

People in the group whose preferred effect of the BAC level is slight buzz tend to increase their BAC. However, the strong influence of 3 on 4 makes 4 change his/her behavior in a way that his/her BAC tends to reach BAC levels in the interval that produces the slight buzz effect. The mutual influences of 4 on 5 and 6 are not strong enough to change significantly their behavior.

In the second case, individuals 5 and 6 influence each other, as shown in Fig. 8(a). Now, the influence of 4 on 5 and 6 is stronger than the one in the first case in Fig. 7(a). The trajectories of the BAC through time in Fig. 8(b) show that individuals 4–6 are affected by the influence of individual 3 on 4, where their BAC tends to increase even though their personal preference is no drinking.

B. Estimation of Group Dynamics From Field Data

The information in [3] contains field data from 1024 people surveyed at 30 different bars. Data collected from each subject include the BAC measurements before entering the bar and after leaving it, duration time in the bar, whether the subject is alone or not, the amount of money available to spend on food, and the amount of money available to spend on alcohol. The subjects reported the level of alcohol intoxication that they desired to reach during the drinking activity by choosing one of the following categories: not drinking, not drinking

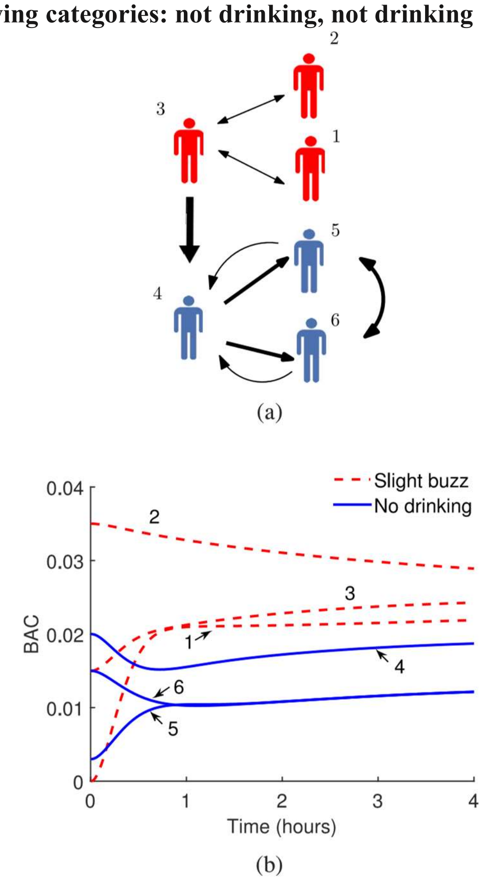


Fig. 8. (a) Influence network in a group where individuals 1–3 have slight buzz as preferred effect of the BAC level, and individuals 4–6 prefer no drinking. The difference of this network with respect to the one in Fig. 7(a) is that now individuals 5 and 6 influence each other, and the influence of 4 on 5 and 6 is stronger. (b) Trajectories of the simulated BAC for each one of the individuals in the group where the influence network is given in (a).

enough to get buzzed, slight buzz, and drunk. In addition to this information, there is observational data that report how crowded the bar was, and whether the subjects were exposed to drinking games and alcohol and food specials. Despite of the large amount of data collected from drinking events at different places, the only available information about the individual's drinking patterns is the BAC measured at the entrance and exit of the bar. We show in this section how we can have a tentative estimation of the trajectory of the BAC level during the drinking event given the available field data and some assumptions on the dynamics of the group.

First, we define the personal preference and environment functions. Since the subjects reported the desired effect of the BAC level that they wanted to reach during the drinking event, we use the preference profiles as shown in Fig. 5(a). The environment function is constructed as the linear combination of influences that either protect against or favor increases in the alcohol level. The amount of money available to spend on food, whether food specials are offered or not, and how crowded the bar was, is information used to construct the component of the environment profile that promotes lower alcohol levels. On the other hand, the presence of drinking games, the amount of money to spend on alcohol, and alcohol specials represent information useful to construct the component of the environment profile that promotes higher BAC. The perception of the BAC rate of change ζ_i in (5) is assumed to be the same for all the individuals.

The only information available about the social interaction is whether the subject is alone or not. Since there are no reported observations of the social interactions between subjects, we assume in our model that there exists a nonobserved social component that allows the subject to achieve the reported final BAC level, given the preferred effect of the BAC level, environment profile, initial BAC level, and the duration time of the drinking activity. Assuming that parameter $b = 0$, for those that reported that were not alone during the drinking event (3) can be rewritten as

$$a_i(t) = -\bar{w}_i(x_i(t) - \bar{x}_i(t)) \qquad (6)$$

where $\bar{w}_i = \sum_{j \in N_i} w_{ij}$ is the total influence strength acting on i, and $\bar{x}_i = (1/\bar{w}_i)\sum_{j \in N_i} w_{ij} x_j$ is the weighted average of the BAC level of those that influence individual i. Here, we assume that $\bar{x}_i$ points to the reported final BAC level. Parameter $\bar{w}_i > 0$ is computed to be the smallest scalar such that the trajectory of the BAC level generated using (3) along with (6) and the previous assumptions reaches the measured final BAC level during the reported time of the drinking activity and starting at the measured initial BAC level. Low values of $\bar{w}_i$ do not imply that there was not social interactions during the drinking event. It implies that, under the assumptions on the parameters of the model and the reported information, the unobserved social pressures are not significantly stronger than other influences to modify the drinking patterns of the individual. The details of the implementation, including the construction of the environment profile, can be found in Section D of the supplementary file.

Fig. 9 shows the results obtained using our model following the methodology and assumptions described above on the data collected *in situ* from individuals that reported drunk as desired effect of BAC level to reach during the drinking event. Using the corresponding preference function as shown in Fig. 5(a), the environment function constructed from the information in the field data, the initial and final BAC, and the time spent in the bar, we show in Fig. 9(a) the estimated trajectories followed by ten randomly selected subjects during the drinking activity (solid lines), and the estimated strength of the social influence on the subjects (thickness of the lines), which is proportional to $\bar{w}_i$ in (6). Note that the thickest line corresponds to the trajectory of a subject that maintains a relatively low level of alcohol with respect to his/her preference of getting drunk. This person reported that he/she did not get food, the bar was not crowded, there were not food specials, and there were alcohol specials and drinking games. Under the assumptions on the parameters of the model and the reported information, the dynamics estimated from the model suggest that there were strong social pressures on the subject that influenced his/her behavior.

Also, we show in Fig. 9(b) a plot of the BAC when the subjects exited the bar versus the BAC before they entered it. The size of the markers is proportional to the estimated strength of the social influence on the individuals. Note that individuals who maintain lower levels of the BAC tend to have larger

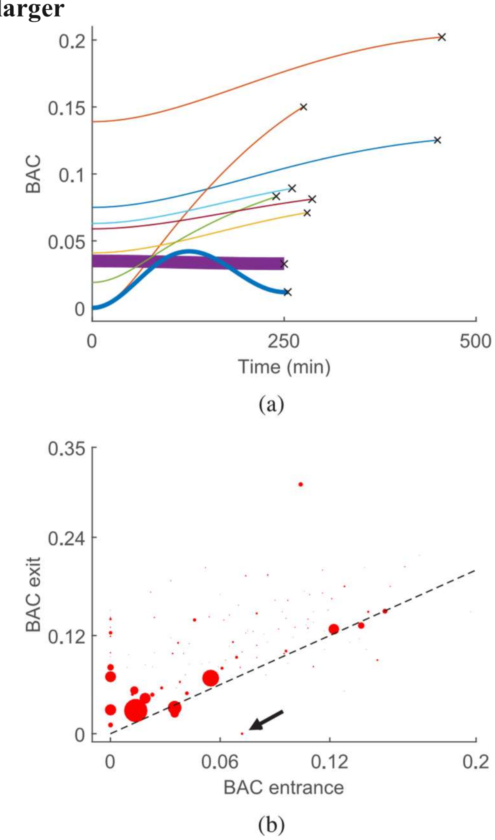


Fig. 9. (a) Estimated trajectories of the BAC provided by the model for ten randomly selected subjects that reported drunk as the preferred level of alcohol intoxication, where symbol "x" marks the BAC measured when the subject exited the bar. (b) Samples of the BAC level when entering the bar (horizontal axis) and after leaving it (vertical axis). The dashed line is a 45° line for reference. The thickness of the lines in (a) and the size of the markers in (b) are proportional to the strength of the social influence acting on the subjects.

social pressures acting on them than those that have larger levels of the BAC. These results are consistent with the fact that this set of subjects had drunk as the desired level of intoxication to reach while they were in the bar, hence additional pressures on the behavior are required to maintain lower values of the BAC. The sample pointed by the arrow corresponds to an individual that was not exposed to drinking games and had little money to spend in alcohol. In this case, the results given by the model suggest that the social pressures were not significantly larger than the other influences to change the dynamics of the individual.

IV. Mathematical Analysis of the Group Dynamics

In the previous section, we provided computer simulations to show the dynamics that the proposed model is able to describe and that are consistent with the observations of groups that are in a alcohol-consumption environment for a given set of parameters and initial conditions. Now, through a mathematical analysis of the model, we study how the model parameters affect all possible trajectories of the BAC level that can be described by the system in (5) given any initial condition. This analysis is based on Lyapunov stability theory [26], which equips us with mathematical tools to derive statements on the dynamics of the group given specific parameter settings. First, we study the case when the influence strength of the social interactions and the environment is zero and the behavior of the individual is driven mainly by his/her desire to reach a specific effect of the BAC level. Theorem 1 shows that each individual reaches the desired effect in the drinking activity when there is no influence of other members of the group and the environment. Then, through Theorem 2, we show how the dynamics of the group are affected when there are social interactions and also environmental pressures on the group. We also discuss the implications of these mathematical results.

A. No Social and Environmental Pressures

In this analysis, we use the concept of asymptotic stability to show that when there are no social and environmental pressures, the individuals achieve their personal preferences in the long term. We start by assuming that f_i^p (personal preference function of individual i) is a strictly convex function that has a unique minimum at x_i^*, for $i = 1,\ldots,n$. Without loss of generality, $f_i^p(x_i^*) = 0$ for every $i \in \{1,\ldots,n\}$. Let $\tilde{x}_i = x_i - x_i^*$ be the relative BAC level of individual i with respect to his/her desired BAC level. Assume that there is no influence from the other members of the group and the environment on the individual. Then, the dynamics of i can be written as

$$\dot{\tilde{x}}_i = v_i$$
$$\dot{v}_i = -\eta_i^p \frac{df_i^p}{dx_i}\left(\tilde{x}_i + x_i^*\right) - \zeta_i v_i, \qquad i = 1,\ldots,n. \tag{7}$$

The following theorem shows that an individual modulates his/her own behavior such that his/her BAC level reaches the desired one when there is no pressures from the social interactions and the environment the group is in, and that the concept of perception of the BAC rate of change affecting the BAC level has to be included in the model to guarantee convergence of the trajectories of the BAC level.

Theorem 1: Consider the dynamics of the individuals in the group characterized in (7). The point $\tilde{x}_i = 0$ and $v_i = 0$ is a globally asymptotically stable equilibrium.

Proof: Let $V_i(\tilde{x}_i, v_i) = \eta_i^p f_i^p(\tilde{x}_i + x_i^*) + (1/2)|v_i|^2$. From the assumptions that $f_i^p(x_i^*) = 0$ and that f_i^p is strictly convex and has a unique minimum at x_i^*, we have that $f_i^p(\tilde{x}_i + x_i^*) > 0$ for all $\tilde{x}_i \neq 0$. Since $|v_i|^2 > 0$ for all $v_i \neq 0$, it implies that $V_i(\tilde{x}_i, v_i) = 0$ only when $\tilde{x}_i = 0$ and $v_i = 0$, and $V_i(\tilde{x}_i, v_i) > 0$ otherwise. We use then V_i as our Lyapunov function candidate. Its derivative with respect to time is

$$\dot{V}_i(\tilde{x}_i, v_i) = \eta_i^p \frac{df_i^p}{dx_i}\left(\tilde{x}_i + x_i^*\right)\dot{\tilde{x}}_i + \frac{1}{2} v_i \dot{v}_i = -\zeta_i |v_i|^2 \le 0$$

where $\dot{V}_i(\tilde{x}_i, v_i)$ is negative semidefinite. However, from (7), we can see that if $v_i(t) = 0$ for a given t, variable $\dot{v}_i(t)$ is not zero unless $\tilde{x}_i(t) = 0$. This is consistent with LaSalle's invariance principle [26, Th. 4.4]. Let $S = \{\tilde{x}, v : \dot{V}(\tilde{x}, v) = 0\}$. From (7), we have that no solution can stay identically in S other than $\tilde{x}_i = 0$ and $v_i = 0$. Therefore, from LaSalle's invariance principle, the solution $\tilde{x}_i = 0$ and $v_i = 0$ is globally asymptotically stable. ■

Remark 1: Since there is no effect of the environment on the group and there is no coupling between its members, the force that drives the individual's behavior is his/her own preferences. Hence, the dynamics of an individual in the drinking group modeled in (7) are the result of an optimization process driven by the cost function f_i^p.

Remark 2: The fact that ζ_i is greater than zero guarantees that the individual approaches the point $\tilde{x}_i = 0$ and $v_i = 0$ as t tends to infinity. The term $-\zeta_i v_i$ in (7) causes a gradual deceleration of the trajectories of $\tilde{x}_i$ as $\tilde{x}_i$ approaches zero, i.e., when x_i approaches x_i^*.

B. Influence of the Environment and Social Interactions on the Group Dynamics

To examine the influence of the environmental and social pressures on the dynamics of the drinking group, we now think of the group as a dynamical system that is a perturbation of the "nominal" system given in (7), where the perturbations are given by the components of the force associated with the environment and social interactions in (3) and (4). In this case, we study the dynamics of the system through the concept of uniform ultimate boundedness [26, Ch. 4.8], which allows us to characterize how the resultant dynamics of the BAC level in the group members deviate from their desired level in terms of the model parameters.

Assume that f^e (environment function) is a strictly convex function and has a unique minimum $x_e \in \mathbb{R}$, and that its derivative is Lipschitz continuous, that is, for any $z,y \in \mathbb{R}$ there exists a constant $M^e > 0$ such that $|(df^e/dx)(z) - (df^e/dx)(y)| \leq M^e|z-y|$. Also, as in Section IV-A, we assume that f_i^p is strictly convex and has a unique minimum at x_i^* with $f_i^p(x_i^*) = 0$, and that its derivative is also Lipschitz continuous with constant $M_i^p > 0$, for every $i = 1,\ldots,n$. Let $x = [x_1,\ldots,x_n]$, $x^* = [x_1^*,\ldots,x_n^*]^\top$, $\tilde{x} = [\tilde{x}_1,\ldots,\tilde{x}_n]$, and $v = [v_1,\ldots,v_n]$. Also, let

$$F^p(x) = \left[\eta_1 \frac{df_1^p}{dx}(x_1),\ldots,\eta_n \frac{df_n^p}{dx}(x_n)\right]$$

$$F_e(x) = \left[\eta_1^e \frac{df^e}{dx}(x_1),\ldots,\eta_n^e \frac{df^e}{dx}(x_n)\right].$$

We define matrix L such that its entry at the ij position is given by

$$[L]_{ij} = \begin{cases} 0 & \text{if } i \neq j \text{ and } j \notin N_i \\ -w_{ij} & \text{if } i \neq j \text{ and } j \in N_i \\ \sum_{l \in N_i} w_{il} & \text{if } i = j \end{cases}$$

where $w_{ij} > 0$ is the strength of the influence of j on i defined in (3). This matrix corresponds to the Laplacian of the graph $\mathcal{G}$ that defines the influence network in the drinking group. Then, the model of the drinking group in (5) can be written as $\dot{\tilde{x}} = v$

$$\dot{v} = -L(\tilde{x} + x^*) - bLv - F_p(\tilde{x} + x^*) - F_e(\tilde{x} + x^*) - Dv \tag{8}$$

where D is a $n \times n$ diagonal matrix defined as $D = \mathrm{diag}(\zeta_1,\ldots,\zeta_n)$. The following theorem shows how the social and environmental pressures affect the behavior of each individual in a way that it deviates from their personal preferences.

Theorem 2: Let the social pressures in the drinking group characterized in (8) be such that the strength of the attractions satisfy

$$b\sum_{j\in N_i} w_{ij} + 2\zeta_i > b\sum_{j\in \mathcal{I}_i} w_{ji}, \quad i = 1,\ldots,n \tag{9}$$

where $\mathcal{I}_i = \{j \in \mathcal{V} : i \in N_j\}$ is the set of all members of the group that are influenced by individual i. Then, the trajectories of the social system in (8) are uniformly ultimately bounded with ultimate bound γ given by

$$\gamma = \beta \max_i \left[\frac{1}{\delta_i\sqrt{2n}} \sum_{j\in N_i} w_{ij}\left|x_i^* - x_j^*\right| + \eta_i^e M^e\left|x_e - x_i^*\right|\right] \tag{10}$$

where $\delta_i = \max[c\theta_i, \sum_{j\in\mathcal{I}_i} w_{ji} + \eta_i^e M^e]$, and constants $\beta > 0$, $M^e > 0$, and $\theta_i \in (0,1)$ for $i = 1,\ldots,n$. Constant c is the minimum eigenvalue of the matrix $D + b/2(L + L^\top)$.

Proof: Let L^o be a $n\times n$ matrix whose ijth entry is given by

$$[L^o]_{ij} = \begin{cases} 0 & \text{if } i \neq j \text{ and } j \notin \mathcal{I}_i \\ -w_{ji} & \text{if } i \neq j \text{ and } j \in \mathcal{I}_i \\ \sum_{l\in\mathcal{I}_i} w_{li} & \text{if } i = j. \end{cases}$$

This corresponds to the column Laplacian matrix of the graph that defines the structure of the influence network $\mathcal{G}$. Let

$$V(\tilde{x},v) = \frac{1}{2}\tilde{x}^\top(L + L^o)\tilde{x} + \sum_{i=1}^{n} \eta_i^p f_i^p(\tilde{x}_i + x_i^*) + \frac{1}{2}v^\top v. \tag{11}$$

From the assumption that f_i^p is strictly convex and has a unique minimum at x_i^* and $f_i^p(x_i^*) = 0$ for all $i = 1,\ldots,n$, we have that $\sum_{i=1}^{n} \eta_i^p f_i^p(\tilde{x}_i + x_i^*)$ is positive definite with respect to $\tilde{x}$. Also, $v^\top v$ is positive definite with respect v. From [34, Lemma 5], we have that $\tilde{x}^\top(L + L^o)\tilde{x} = \sum_{i=1}^{n}\sum_{j\in N_i} w_{ij}|\tilde{x}_j - \tilde{x}_i|^2$ is positive semidefinite with respect to $\tilde{x}$. Hence, $V(\tilde{x},v) = 0$ only when $\tilde{x} = 0$ and $v = 0$, and

$V(\tilde{x},v) > 0$ otherwise. We select then V as our Lyapunov function candidate. The derivative of V with respect to time is

$$\dot{V}(\tilde{x},v) = -v^\top (D+bL)v + v^\top \left(L^o\tilde{x} - Lx^* - \sum_{i=1}^{n} v_i^\top \eta_i^e \frac{df^e}{dx}(\tilde{x}_i + x_i^*)\right)$$

$$v_i^\top \left[\sum_{N_i} w_{ij}\left(x_i^* - x_j^*\right)\right.$$

$$= -v^\top (D+bL)v - \sum_{i=1}^{n} \sum_{j\in} w\left(\; -\tilde{x}_j\right) + \eta_i^e \frac{df^e}{dx}\left(\tilde{x}_i + x_i^*\right) + \sum_{j\in_i} w_{ji}\tilde{x}_i \Big] \quad (12)$$

In (12), term $v(D+bL)v$ is positive definite with respect to v if and only if the eigenvalues of the matrix $D+b/2(L+L)$ are positive. From Gershgorin's circle theorem [35, Th. 7.2.1], every eigenvalue λ of this matrix is in the region

$$\left|\lambda - b\sum_{j\in N_i} w_{ij} - \zeta_i\right| \le \frac{b}{2}\left[\sum_{j\in N_i} w_{ij} + \sum_{j\in\Gamma_i} w_{ji}\right].$$

From the assumption (9), we have that all the eigenvalues of this matrix are positive. Let $c = \lambda_{\min}[D+b/2(L+L)]$ be the minimum eigenvalue of the matrix $D+b/2(L+L)$. Then, using the fact that $v(D+bL)v = v^\top(D+b/2(L+L)v$, and from Rayleigh quotient [36, Th. 10.13], we have that $-v^\top(D+bL)v \le -c\sum_{i=1}^{n}|v_i|^2$. Also, since (df_e/dx) is assumed to be Lipschitz continuous, we know that there exists a constant $M^e > 0$ such that $|(df^e/dx)(\tilde{x}_i + x_e) - (df/dx)(x_e)_{*e}| = |(df_e/dx)(\tilde{x}_i + x_e)| \le M_e|\tilde{x}_i - x_{ei*}|$, where $x_{ei} = x_e - x_i$. Using this result in (12), we obtain

$$\dot{V}(\tilde{x},v) \le \sum_{i=1}^{n}\Big[-c|v_i|^2 + |v_i|\left(\sum_{j\in N_i} w_{ij}\left|x_i^* - x_j^*\right| - \tilde{x}_j\right| + \eta_i^e M^e\left|\tilde{x}_i - x_{ei}^*\right|\Big) \Big] + \sum_{j\in_i} w_{ji}|\tilde{x}_i| . \quad (13)$$

For a constant $\theta_i \in (0,1)$, we have that $-c|v_i|^2 = -c(1-\theta_i)|v_i|^2 - c\theta_i|v_i|^2$. Then, we can rewrite the inequality in (13) as

$$\dot{V}(\tilde{x},v) \le \sum_{i=1}^{n} -c(1-\theta_i)|v_i|^2 \quad \text{for all}$$

$$c\theta_i|v_i| - \left(\sum_{j\in\Gamma_i} w_{ji} + \eta_i^e M^e\right)|\tilde{x}_i| - \sum_{j\in\Gamma_i} w_{ji}|\tilde{x}_j|$$

$$\ge \sum_{j\in N_i} w_{ij}\left|x_i^* - x_j^*\right| + \eta_i^e M^e\left|x_e - x_i^*\right|, \quad i = 1,\ldots,n. \quad (14)$$

Equation (14) can be written in a compact yet more conservative way

$$\dot{V}(\tilde{x},v) \le \sum_{i=1}^{n} -c(1-\theta_i)|v_i|^2$$

$$\text{for all } \left\|\left[\tilde{x},v\right]^\top\right\| \ge \mu \quad (15)$$

where

$$\mu = \max_i \left[\frac{\sqrt{2n}}{\delta_i}\left(\sum_{j\in N_i} w_{ij}\left|x_i^* - x_j^*\right| + \eta_i^e M^e\left|x_e - x_i^*\right|\right)\right] \quad (16)$$

and

$$\delta_i = \max\left[c\theta_i, \; \sum_{j\in\Gamma_i} w_{ji} + \eta_i^e M^e\right].$$

This expression was obtained using the norm inequality [26, p. 648]

$$\sum_{i=1}^{n} |\tilde{x}_i| + |v_i| = \left\|\left[\tilde{x},v\right]^\top\right\|_1 \le \sqrt{2n}\left\|\left[\tilde{x},v\right]\right\|_2.$$

Equation (15) indicates that $\dot{V}$ is negative semidefinite for all $\|[\tilde{x},v]^\top\|_2 \ge \mu$. However, similar to the proof of Theorem 1, from (8) we know that if $v(t) = 0$ for a given t,

variable $\dot{v}(t)$ is not zero unless additional conditions on both $\tilde{x}(t)$ and $v(t)$ are satisfied. Hence, $\dot{V}$ is negative as long as the bound in (15) is satisfied. Thus, the trajectories of the social system in (8) are uniformly ultimately bounded. From now on, unless it is indicated, the vector norm $\|\cdot\|$ corresponds to the L_2 vector norm.

To compute the ultimate bound, we need to find strictly increasing functions α_1 and α_2, with $\alpha_1(0) = 0$ and $\alpha_2(0) = 0$, such that

$$\alpha_1\left(\left\|\left[\tilde{x}, v\right]^\top\right\|\right) \le V(\tilde{x}, v) \le \alpha_2\left(\left\|\left[\tilde{x}, v\right]^\top\right\|\right).$$

According to [26, Th. 4.18], the ultimate bound will be given by $\gamma = \alpha_1^{-1}(\alpha_2(\mu))$. From (11), we know that $V(\tilde{x}, v)$ satisfies

$$\begin{aligned} V(\tilde{x}, v) &\ge \frac{1}{2}\lambda_{\min}\left[L + L^o\right]\|\tilde{x}\|^2 + \frac{1}{2}\|v\|^2 \\ &\ge \beta_1\left[\|\tilde{x}\|^2 + \|v\|^2\right] = \alpha_1\left(\left\|\left[\tilde{x}, v\right]^\top\right\|\right) \end{aligned} \quad (17)$$

where $\beta_1 = \min\{(1/2)\lambda_{\min}[L + L^o], (1/2)\}$. To find the upper bound of $V(\tilde{x}, v)$, we use the assumption of Lipschitz continuity on (df_i^p/dx) for $i = 1,\dots,n$. Here, from [37, Proposition A.24], we have that there exists a constant $M_i^p > 0$ such that f_i^p satisfies $f_i^p(\tilde{x}_i + x_i^*) \le M_i^p |\tilde{x}_i|^2/2$. Using this property, we obtain

$$V(\tilde{x}, v) \le \frac{1}{2}\lambda_{\max}\left[L + L^o\right]\|\tilde{x}\|^2 + \sum_{i=1}^{n} \frac{\eta_i^p M_i^p}{2}|\tilde{x}_i|^2 + \frac{1}{2}\|v\|^2. \quad (18)$$

Equation (18) then can be bounded as

$$\begin{aligned} V(\tilde{x}, v) &\le \sum_{i=1}^{n} \beta_{2i}\left[|\tilde{x}_i|^2 + |v_i|^2\right] \\ &\le \beta_2\left\|\left[\tilde{x}, v\right]^\top\right\|^2 = \alpha_2\left(\left\|\left[\tilde{x}, v\right]^\top\right\|\right) \end{aligned} \quad (19)$$

where

$$\beta_{2i} = \max\left[\frac{1}{2}\lambda_{\max}\left[L + L^o\right], \frac{\eta_i^p M_i^p}{2}, \frac{1}{2}\right]$$

and

$$\gamma = \alpha_1^{-1}(\alpha_2(\mu)) = \sqrt{\frac{\alpha_2(\mu)}{\beta_1}} = \sqrt{\frac{\beta_2}{\beta_1}}\mu = \beta\mu$$

$$\beta_2 = \max_{i=1,\dots,n} \beta_{2i}.$$

From (17) and (19), we can obtain an expression for the ultimate bound where μ is defined in (16), and $\beta = \sqrt{\beta_2/\beta_1}$. ■

Remark 3: Theorem 2 indicates that the social interactions and the environment add dissipative elements in the dynamics that make the individuals deviate from their personal preferences. Note that the ultimate bound in (10) mainly depends on the social influence and environment parameters.

Remark 4: If the ultimate bound γ defined in (10) is zero, it means that every individual follows his/her personal preferences. There are two cases when this situation happens. First, when there is no social interaction and no influence of the environment (i.e., $w_{ij} = 0$, $\eta_i^p =$ for all $i \in \mathcal{V}$, and $j \in N_i$), which is the case described in Theorem 1. The other case is when $x_i^* = x_j^*$, and $x_i^* = x_e$, for all $i \in \mathcal{V}$, $j \in N_i$). It means that, given the initial conditions of the variables x and v, in the long term all the individuals will reach their desired BAC level even though there are some force components that pressure the individuals to have the same BAC and its rate of change during the drinking activity.

Remark 5: The ultimate bound in (10) tends to zero as the strength of the social interactions and/or the difference between the desired positions of the individuals decrease.

Remark 6: In Theorem 2, Lipschitz continuity in the gradient of the personal preference and environment functions is a reasonable assumption that implies some convenient properties for the functions that allow us to obtain results that are easily interpretable.

Remark 7: Note that there are no assumptions on the connectivity of the structure of the influence network $\mathcal{G}$. The only related assumption is the one in (9), which guarantees that, in the long term, the rate of change in BAC level of the individuals v will be zero. There are special cases when this assumption can be satisfied: when the network $\mathcal{G}$ is balanced, that is,

$\sum_{j\in N_i} w_{ij} = \sum_{j\in_i} w_{ji}$ for all $i \in \mathcal{V}$ (i.e., the total strength of the influence of individuals in the group on i equals the total strength of the influence of individual i on others); and when there are no attractions on the rate of change in BAC in the group, that is, $b = 0$.

The results given in Theorem 1 suggest that there must exist an influence acting on the individual's behavior that depends on his/her perception on how quickly his/her own BAC level is changing and modulates the dynamics of the

BAC accordingly. Also, the ultimate bound provided in Theorem 2 shows that the compound action of the environment (with strength proportional to η_i^e) and social influences (with strength proportional to the w_{ij}) can have a significant impact on the BAC level trajectories at the individual level, as expected. It is important to note that the design of interventions has focused mainly on affecting the conditions in the environment through individual incentives to prevent problems like heavy alcohol consumption. However, as shown in the ultimate bound from Theorem 2, an individual's drinking behavior can be impacted by having either a large influence coming from the environment, or small social influences that add up together. This observation suggests that intervention designs at the social level could lead to promising outcomes in terms of prevention of unhealthy drinking behaviors. For example, a recent study has shown that using social pressures instead of individual incentives to increase physical activity levels in a community provided significantly better results [10, Ch. 4].

V. Conclusion

Methodologically, this paper illustrates how field data and computational and mathematical modeling complement each other. In our original field studies, we did not have the ability to directly measure or model the influence of groups on drinking behavior. The combination of these approaches represents a way to maximize the data collected in large studies. Furthermore, measuring dynamical processes using traditional social science methods is often not possible or extremely difficult [38]. As part of the cyclic process in Fig. 1, our model along with the mathematical analysis and simulations presented above will help us refine our future field studies, especially as they relate to the interplay between individual, group, and environment and the relationship to alcohol intoxication. Also, as recent technological advances improve our ability to collect real-time data, we will better inform the empirical specification of the proposed model [5].

Theoretically, through the model presented above and its analysis through stability theory and simulations, we hope to inform our understanding of group dynamics as they relate to drinking behavior. Traditional social psychological models [15] have given us the foundation from which to build more sophisticated complex dynamical models. Understanding group influence in the context of environment, network relationships, and individual preference—taking into account for the influence of psychoactive substances—affords a richer etiological understanding of real world phenomena like drinking behavior. In turn, understanding the influences at different levels, how environment can moderate personal in-group influence for example, may ultimately help guide applied preventive solutions to problems like heavy alcohol consumption and the problems that flow from that consumption.

On the modeling side of this paper, our next steps include developing models that incorporate additional information about the individual, such as gender or weight. In this way, the model can be coupled with Wegner's equations of alcohol content [39] to characterize the trajectories of the BAC given the number and type of drinks per time unit. On the empirical side of this paper, we are designing a realtime data collection process at drinking events that includes measurements of variables at the individual, group, and environmental level [19], [20], [22]. Our aim is to study these data using tools from both statistics and dynamical system theory [26]. We plan to do system identification [40] to find the parameters that allow the model to have the closest approximation to the measured behaviors. Depending on the results, we will validate and improve our hypotheses on the mechanisms that drive behavior during the drinking event. We hope that this subsequent round of model and field validations will have contributed to our understanding sufficient to engage in a series of interventions at the event level.

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Luis Felipe Giraldo is currently pursuing the Ph.D. degree in electrical and computer engineering with Ohio State University, Columbus, OH, USA.

His current research interests include modeling and analysis of linear and nonlinear dynamical systems, pattern recognition, and signal processing.

Kevin M. Passino (S'79–M'90–SM'96–F'04) received the Ph.D. degree in electrical engineering from the University of Notre Dame, Notre Dame, IN, USA, in 1989.

He is a Professor of Electrical and Computer Engineering and the Director of the Humanitarian Engineering Center with Ohio State University, Columbus, OH, USA. He has published a book entitled *Humanitarian Engineering: Creating Technologies that Help People, Edition 2* (Columbus: Bede, 2015).

John D. Clapp received the Ph.D. degree in social work from Ohio State University, Columbus, OH, USA, in 1995.

He is a Professor and the Associate Dean of Research and Faculty Development with the College of Social Work, Ohio State University, where he also serves as the Director of the Higher Education Center for Alcohol and Drug Misuse Prevention and Recovery. He is the Editor-in-Chief of the *International Journal of Alcohol and Drug Research*.