Fuzzy Learning Control for Antiskid Braking Systems

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Abstract—Although antiskid braking systems (ABS) are designed to optimize braking effectiveness while maintaining steerability, their performance often degrades for harsh road conditions (e.g., icy/snowy roads). In this brief paper we introduce the idea of using the fuzzy model reference learning control (FMRLC) technique [1] for maintaining adequate performance even under such adverse road conditions. This controller utilizes a learning mechanism which observes the plant outputs and adjusts the rules in a direct fuzzy controller so that the overall system behaves like a "reference model" which characterizes the desired behavior. The performance of the FMRLC-based ABS is demonstrated by simulation for various road conditions (wet asphalt, icy) and transitions between such conditions (e.g., when emergency braking occurs and the road switches from wet to icy or vice versa).

I. INTRODUCTION

ANTISKID braking systems (ABS) present a challenging control problem since there can be significant brake/automotive system parameter variations (e.g., due to brake pad coefficient of friction changes or road slope variations) and environmental influences (e.g., due to adverse road conditions). While conventional control approaches [2]-[4] and even direct fuzzy/knowledge based approaches [5]-[8] have been successfully implemented, their performance may still degrade when adverse road conditions are encountered. The basic reason for this performance degradation is that the control algorithms have limited ability to learn how to compensate for the wide variety of road conditions that exist. In this paper we will investigate the role that learning controllers can take in enabling ABS to compensate for adverse road conditions.

A "learning system" possesses the capability to improve its performance over time by interaction with its environment. A learning control system is designed so that its "learning controller" has the ability to improve the performance of the closed-loop system by generating command inputs to the plant and utilizing feedback information from the plant. The learning mechanism in the fuzzy model reference learning control (FMRLC) system that we design for the ABS will monitor the performance of a fuzzy controller and tune it to adapt to adverse road conditions as they are encountered. This FMRLC was first introduced in [1], [9], [10] and it grew from ideas in linguistic self-organizing control (SOC) [11] and conventional model reference adaptive control (MRAC) [12]. In fact, it has provided significant improvements over the SOC approach for enhanced performance feedback and knowledge base modification [1], [9] and has compared favorably to the MRAC for a ship steering application [13].

In this paper we illustrate that the FMRLC provides an effective solution to the problem of compensating for certain adverse road conditions. We begin by describing the ABS under consideration. Next, we illustrate the FMRLC performance for a vehicle during braking on dry asphalt, wet asphalt, and an icy surface. Finally, we study FMRLC performance for transitions between such road conditions. In particular, we study braking effectiveness when there are transitions between icy and wet road surfaces. This paper is an expanded version of the work reported in [14].

In Section II we will overview the FMRLC technique. In Section III we describe the ABS problem while in Section IV we provide simulation results that give an initial assessment of the performance of the FMRLC for ABS. Section V provides some concluding remarks.

II. FUZZY MODEL REFERENCE LEARNING CONTROL

The FMRLC, which is shown in Fig. 1, utilizes a learning mechanism that: 1) observes data from a fuzzy control system [i.e., \( y_r(kT) \) and \( u(kT) \)]; 2) characterizes its current performance; and 3) automatically synthesizes and/or adjusts the fuzzy controller so that some prespecified performance objectives are met. These performance objectives are characterized via the reference model shown in Fig. 1. In an analogous manner to conventional MRAC where conventional controllers are adjusted, the learning mechanism seeks to adjust the fuzzy controller so that the closed-loop system [the map from \( y_r(kT) \) to \( y(kT) \)] acts like a prespecified reference model [the map from \( y_r(kT) \) to \( y_m(kT) \)]. Next we describe each component of the FMRLC in more detail.

A. The Fuzzy Controller

The process in Fig. 1 is assumed to have \( r \) inputs denoted by the \( r \)-dimensional vector \( u(kT) = [u_1(kT) \cdots u_r(kT)]^T \) (\( T \) is the sample period) and \( s \) outputs denoted by the \( s \)-dimensional
vector $y(kT) = [y_1(kT) \cdots y_n(kT)]^T$. Most often the inputs to the fuzzy controller are generated via some function of the plant output $y(kT)$ and reference input $y_r(kT)$. Fig. 1 shows a special case of such a map that was found useful in many applications. The inputs to the fuzzy controller are the error $e(kT) = [e_1(kT) \cdots e_n(kT)]^T$ and change in error $c(kT) = [c_1(kT) \cdots c_n(kT)]^T$ defined as

$$e(kT) = y_r(kT) - y(kT),$$

$$c(kT) = \frac{e(kT) - e(kT - T)}{T},$$

respectively, where $y_r(kT) = [y_{r1}(kT) \cdots y_{rn}(kT)]^T$ denotes the desired process output.

In fuzzy control theory, the range of values for a given controller input or output is often called the “universe of discourse” [15]. Often, for greater flexibility in fuzzy controller implementation, the universes of discourse for each process input are “normalized” to the interval $[-1, 1]$ by means of constant scaling factors. For our fuzzy controller design, the gains $g_e, g_c, g_r$ were employed to normalize the universe of discourse for the error $e(kT)$, change in error $c(kT)$, and controller output $u(kT)$, respectively (e.g.,

$$g_e = [g_{e1} \cdots g_{en}]^T$$

so that $g_e e(kT)$ is an input to the fuzzy controller). The gains $g_e$ are chosen so that the range of values of $g_e e(kT)$ lie on $[-1, 1]$ and $g_c$ is chosen by using the allowed range of inputs to the plant in a similar way. The gains $g_r$ are determined by experimenting with various inputs to the system to determine the normal range of values for which the desired values of $c(kT)$ will take on; then $g_e$ is chosen so that this range of values is scaled to $[-1, 1]$.

We utilize a multiple-input single-output (MISO) fuzzy controllers, one for each process input $u_n$ (equivalent to using one MIMO controller). The knowledge base for the fuzzy controller associated with the nth process input is generated from IF-THEN control rules of the form:

$$\text{If } \tilde{e}_1 \text{ is } E^b_1 \text{ and } \cdots \text{ and } \tilde{e}_n \text{ is } E^b_n \text{ and }$$

$$\text{Then } \tilde{u}_n \text{ is } U^b_1 \ldots \cdot U^b_n$$

where $\tilde{e}_n$ and $\tilde{c}_n$ denote the linguistic variables associated with controller inputs $e_n$ and $c_n$, respectively. $u_n$ denotes the linguistic variable associated with the controller output $u_n$.

$E^b_1$, $E^b_2$, $E^b_3$, $U^b_1$, $U^b_2$, $U^b_3$ denote the bth linguistic value associated with $e_n$ and $c_n$, respectively, and $U^b_1, \ldots, U^b_n$ denotes the consequent linguistic value associated with $u_n$. The above control rule may be quantified by utilizing fuzzy set theory to obtain a fuzzy implication of the form:

$$\text{If } E^b_1, \ldots, E^b_n \text{ and } C^m_1 \text{ and } \cdots \text{ and } C^m_n \text{ then } U^b_1 \ldots \cdot U^b_n,$$

where $E^b_1, C^m_1, U^b_1, \ldots, U^b_n$ denote the fuzzy sets that quantify the linguistic statements “$e_n$ is $E^b_n$” “$c_n$ is $C^m_n$” and “$u_n$ is $U^b_n$” respectively. This fuzzy implication can be represented by a fuzzy relation

$$R^b_1 \ldots \cdot R^b_n = (E^b_1 \times \cdots \times E^b_n) \times (C^m_1 \times \cdots \times C^m_n) \times U^b_1 \ldots \cdot U^b_n.$$

A set of such rules forms the “rule-base” which characterizes how to control a dynamical system. We use triangular membership functions for the input and output (normalized) universes of discourse, Zadeh’s compositional rule of inference, and the standard center-of-gravity (COG) defuzzification technique [15].

B. The Reference Model

The reference model provides a capability for quantifying the desired performance. In general, the reference model may be any type of dynamical system (linear or nonlinear, time-invariant or time-varying, discrete or continuous time, etc.). The performance of the overall system is computed with respect to the reference model by generating an error signal

$$y_e(kT) = [y_{e1} \cdots y_{en}]^T$$

where

$$y_e(kT) = y_r(kT) - y(kT).$$

Given that the reference model characterizes design criteria such as rise time and overshoot and the input to the reference model is the reference input $y_r(kT)$, the desired performance of the controlled process is achieved if the learning mechanism forces $y_e(kT)$ to remain very small for all time; hence, the error $y_e(kT)$ provides a characterization of the extent to which the desired performance is achieved at time $kT$. If the performance is met ($y_e(kT) \approx 0$) then the learning mechanism will not make significant modifications to the fuzzy controller. On the other hand if $y_e(kT)$ is big, the desired performance is not achieved and the learning mechanism must adjust the fuzzy controller. Next we describe the operation of the learning mechanism.

C. The Learning Mechanism

As previously mentioned, the learning mechanism performs the function of modifying the knowledge base of a direct fuzzy controller so that the closed-loop system behaves like the reference model. These knowledge base modifications are made by observing data from the controlled process, the reference model, and the fuzzy controller. The learning mechanism consists of two parts: a fuzzy inverse model and a knowledge base modifier. The fuzzy inverse model performs
the function of mapping $y_i(kT)$ (representing the deviation from the desired behavior), to changes in the process inputs $p = [p_1 \cdots p_T]^T$ that are necessary to force $y_i(kT)$ to zero. The knowledge base modifier performs the function of modifying the fuzzy controller’s knowledge base to affect the needed changes in the process inputs. More details of this process are discussed next.

The **fuzzy inverse model** was developed in [1], [9] by investigating methods to alleviate the problems with using the inverse process model in the linguistic SOC framework of Procyk and Mamdani [11]. Procyk and Mamdani’s use of the inverse process model depended on the use of an explicit mathematical model of the process (and its inverse) and ultimately on restrictive assumptions about the underlying physical process (which cause significant difficulties in applying their approach). Using the fact that most often a control engineer will know how to roughly characterize the inverse model of the plant, the authors in [1], [9] introduce the idea of using a fuzzy system to map $y_i(kT)$ and possibly functions of $y_i(kT)$ (or process operating conditions), to the necessary changes in the process inputs $p(kT)$. This map is called the “fuzzy inverse model” since information about the plant inverse dynamics is used in its specification. Note that similar to the fuzzy controller, the fuzzy inverse model shown in Fig. 1 contains normalizing scaling factors, namely $g_{y_e}, g_{y_a}$, and $g_p$, for each universe of discourse.

Given that $y_e, y_a$, and $y_e, y_a$ are inputs to the fuzzy inverse model, the knowledge base for the fuzzy inverse model associated with the $n$th process input is generated from fuzzy implications of the form

$$IF Y_e^b and \cdots and Y_e^a and Y_a^l and \cdots and Y_a^m \hspace{1cm} THEN P^b_{i,n} \cdots k,l, \cdots, m$$

where $Y_e^b$ and $Y_e^a$ denote the $b$th fuzzy set for the error $y_e$, and change in error $y_a$, respectively, associated with the $n$th process output and $P^b_{i,n} \cdots k,l, \cdots, m$ denotes the consequent fuzzy set for this rule describing the necessary change in the $n$th process input. As with the fuzzy controller we utilize triangular membership functions for both the input and output universes of discourse, Zadeh’s compositional rule of inference, and COG defuzzification.

The **knowledge base modifier** for the FMRLC also grew from research performed on the linguistic SOC [11], [1], [9]. In the linguistic SOC framework, knowledge base modification was performed on the overall fuzzy relation ($R_n = U_j, \cdots k, l, \cdots, m P^b_j, \cdots k, l, \cdots, m$) used to implement the fuzzy controller. However, this method of knowledge base modification can be computationally complex due to the fact that $R_n$ is generally a very large array. In [1], [9] the authors presented a new knowledge base modification algorithm which increases computational efficiency by modifying the membership functions of consequent fuzzy sets $U^b_j, \cdots k, l, \cdots, m$ rather than the fuzzy relation array $R_n$.

Given the information about the necessary changes in the input as expressed by the vector $p(kT)$, the knowledge base modifier changes the knowledge base of the fuzzy controller so that the previously applied control action will be modified by the amount $p(kT)$. Therefore, consider the previously computed control action $u(kT - T)$, which contributed to the present good/bad system performance. Note that $e(kT - T)$ and $e(kT - T)$ would have been the process error and change in error, respectively, at that time. By modifying the fuzzy controller’s knowledge base we may force the fuzzy controller to produce a desired output $u(kT - T) + p(kT)$.

Assume that only symmetric membership functions are defined for the fuzzy controller’s output so that $c_{i,n}^b, \cdots k, l, \cdots, m$ denotes the center value of the membership function associated with the fuzzy set $U^b_j, \cdots k, l, \cdots, m$. Knowledge base modification is performed by shifting centers of the membership functions of the fuzzy sets $U^b_j, \cdots k, l, \cdots, m$ which are associated with the fuzzy implications that contributed to the previous control action $u(kT - T)$. This modification involves shifting these membership functions by an amount specified by $p(kT) = [p_1(kT) \cdots p_T(kT)]^T$ so that $c_{i,n}^b, \cdots k, l, \cdots, m(kT - T) + p_n(kT)$. (5)

The degree of contribution for a particular fuzzy implication whose fuzzy relation is denoted $R^b_{i,n} \cdots k, l, \cdots, m$ is determined by its “activation level,” defined by

$$\delta^b_{y_e, \cdots k, l, \cdots, m}(t) = \min\{\mu_{C^b_{1,1}}(c_1(t)), \cdots, \mu_{C^b_{2,n}}(c_n(t))\},$$

(6)

where $\mu_{C^b_{i,j}}(c_i(t))$ denotes the membership function of the fuzzy set $A$. Only those fuzzy implications $R^b_{i,n} \cdots k, l, \cdots, m(kT - T)$ whose activation level $\delta^b_{y_e, \cdots k, l, \cdots, m}(kT - T) > 0$ are modified; all others remain unchanged. It is important to note that our rule-base modification procedure implements a form of **local** learning and hence utilizes memory. In other words, different parts of the rule-base are “filled in” based on different operating conditions for the system, and when one area of the rule-base is updated, other rules are not affected. Hence, the controller adapts to new situations and also remembers how it has adapted to past situations. This justifies the use of the term “learning” rather than “adaptive” (for more details on this point see [16], [1], [9]).

### D. Design Procedure

Note that although it is often not highlighted, most learning/adaptive control approaches assume that an initial controller structure and parameters are given (e.g., initial gains must be known *a priori* in adaptive control approaches). As such initial parameters can impact the overall performance, in what follows we provide a procedure to pick such initial parameters (i.e., the normalizing gains) for the FMRLC.

1) Select the controller gains $g_p$ associated with the desired output change $y_e(kT)$ such that each universe of discourse is mapped to the interval $[-1, 1]$.

2) Choose the controller gain $g_p$ to be the same as for the fuzzy controller output gain $g_u$. This will allow $p_i(kT)$ to take on values as large as the largest possible inputs $u_i(kT)$.

3) Assign the numerical value 0 to the scaling factors associated with the changes in the desired output changes (i.e., all elements of $g_u$ are set equal to 0).
4) Apply a step input to the process which is of a magnitude that may be typical for the process during normal operation. Observe the process response and the reference model response.

5) Three cases:
   a) If there exist unacceptable oscillations in a given process output response about the reference model response, then increase the associated element of $g_y$. Go to step 4).
   b) If a given process output response is unable to “keep up” with the reference model response, then decrease the associated element of $g_y$. Go to step 4).
   c) If the process response is acceptable with respect to the reference model response, then the controller design is completed.

For the application presented in this paper, the above gain selection procedure has proven very successful. However, given that the procedure is a result of simulation experience with the FMRLC rather than strict mathematical analysis, it is possible that it will not work for all processes. For some applications (although none of the ones studied in [1], [9], [13], [10]), the procedure may result in an unstable process. In such situations, it may be necessary to modify other controller parameters such as the controller sampling period $T$ or the number of fuzzy controller rules. Clearly, the stability analysis of the FMRLC is an important research direction. Additional research directions, a discussion of the limitations of the FMRLC, and a comparative analysis of FMRLC and MRAC are provided in [1], [9], and [13].

III. ANTISKID BRAKING SYSTEMS

The objective of the FMRLC-based ABS system is to regulate wheel slip to maximize the coefficient of friction between the tire and road for any given road surface. In general, the coefficient of friction $\mu$ during a braking operation can be described as a function of slip $\lambda$, which for a braking operation is defined as

$$\lambda(t) = \frac{V_v(t)}{R_w} - \frac{\omega_w(t)}{R_w}$$

where $\omega_w(t)$ is the angular velocity of the wheel, $V_v(t)$ is the velocity of the vehicle, and $R_w$ is the radius of the tire. Since the term $(V_v(t)/R_w)$ is the angular velocity of the vehicle with respect to the tire angular velocity, we will sometimes denote this quantity by $\omega_v(t)$. The braking coefficient of friction as a function of slip $\mu(\lambda)$ was measured in [17], [18]. The results of these experiments were approximated for dry asphalt, wet asphalt, and ice as shown in Fig. 2. As one would expect, the braking coefficient of friction is largest for dry asphalt, slightly reduced for wet asphalt, and significantly reduced for ice. From (7) and Fig. 2 observe that 0 % slip represents the free rolling wheel condition ($\omega_w = \omega_v$ and $\mu(\lambda) = 0$) while 100 % slip corresponds to a wheel that is locked ($\omega_w = 0$).

From Fig. 2 we see that for the three road conditions shown, $\mu(\lambda)$ is maximized for $\lambda \approx 20$ %. Although an ideal situation would be to maximize the road/tire friction regardless of $\lambda$, for this study we seek to regulate slip to 20 % to maximize the coefficient of friction between the tire and the road. While the “target slip” value is a subject of debate (many ABS engineers use a more conservative 15 to ensure stable operation), our algorithm will perform similarly if designed for another target value. Regardless of the design target chosen, the important point is that during normal vehicle operation, the road conditions are constantly changing. Since the road surface directly affects the braking characteristic, a controller design which compensates for all possible types of road conditions is difficult (especially for transitions between road conditions).

A greatly simplified model for a vehicle, a single wheel, and its braking system was employed for this research (we ignore actuator dynamics for this initial study). The process model contains linearized vehicle dynamics and one-wheel rotational dynamics, where wind resistance effects and all the vertical dynamics associated with the suspension system are assumed negligible. The differential equation which describes the motion of the wheels can be determined by summing the rotational torques which are applied to the wheel; hence

$$\dot{\omega}_w(t) = \frac{1}{J_w} [-T_b(t) - \omega_w(t)B_w + T_f(t)]$$

where $J_w$ is the rotational inertia of the wheel, $B_w$ is the viscous friction of the wheel, $T_b(t)$ is the braking torque (in N-m), and $T_f(t)$ is the torque generated due to slip between the wheel and the road surface. In general, $T_f(t)$ is a function of the force $F_f(t)$ exerted between the wheel and the road surface, or $T_f(t) = R_wF_f(t)$, where $R_w$ is the radius of the wheel. The vehicle dynamics are determined by summing the total forces applied to the vehicle during a normal braking
operation to obtain

\[ \dot{V}_v(t) = -\frac{1}{M_v} [F_f(t) + B_v V_v(t) + F_\theta(\theta)] \]  

(9)

where \( M_v \) is the mass of the vehicle, \( B_v \) is the vehicle viscous friction, \( g \) is the gravitational acceleration constant, \( F_\theta(\theta) \) is the force applied to the car which results from a vertical gradient in the road so that \( F_\theta(\theta) = M_v g \sin(\theta(t)) \) where \( \theta \) is the angle of inclination of the road. The force \( F_f(t) \) is generally expressed as a function of the coefficient of friction and the normal force on the wheel, or \( F_f(t) = \mu(\theta) N_v(\theta) \) where \( N_v(\theta) \) is the normal force applied to the tire. For this model we assume that the vehicle has four wheels and the weight of the vehicle is evenly distributed among these wheels. As a result, the normal force \( N_v(\theta) \) may be expressed by \( N_v(\theta) = \left(M_v g/4 \right) \cos(\theta(t)) \).

The braking system parameters used in this study are vehicle mass \( M_v = 4 \times 342 \text{ kg} \), viscous friction associated with the linear motion of the vehicle \( B_v = 6 \text{ N} \cdot \text{s} \), rotational inertia of the wheel \( J_w = 1.13 \text{ N} \cdot \text{m} \cdot \text{s}^2 \), rolling radius of the wheel \( R_w = 0.33 \text{ m} \), viscous friction associated with the motion of the wheel \( B_w = 4 \text{ N} \cdot \text{s} \), and \( g = 9.8 \text{ m} \cdot \text{s}^2 \) [4]. Since slip is the controlled parameter of the braking system, we desire to measure this quantity. However, currently it is difficult to accurately measure slip directly, so an estimation scheme is necessary. We will assume that sensors for measuring vehicle acceleration and wheel speed are available for estimating slip as is done in [5], [7]. Equation (7) may be rewritten to obtain

\[ \omega_w(t) = (1 - \lambda(t)) \omega_v(t). \]  

(10)

Taking the time derivative of (10) yields

\[ \dot{\omega}_w(t) = (1 - \lambda(t)) \dot{\omega}_v(t) - \dot{\lambda}(t) \omega_v(t) \]  

(11)

where \( \dot{\omega}_w(t) \) is related to the vehicle linear acceleration \( a_v(t) \) by

\[ \dot{\omega}_w(t) = \frac{\dot{V}_v(t)}{R_w} = \frac{a_v(t)}{R_w}. \]  

(12)

Using (12) and the fact that \( \omega_v = (V_v/R_w) \), we obtain

\[ \dot{\omega}_w(t) = (1 - \lambda(t)) \frac{a_v(t)}{R_w} - \dot{\lambda}(t) \frac{V_v(t)}{R_w}. \]  

(13)

Thus, by rearranging (11) we can solve for the wheel slip derivative \( \dot{\lambda}(t) \) which yields

\[ \dot{\lambda}(t) = \frac{1 - \lambda(t)}{V_v(t)} a_v(t) - \left( \frac{R_w}{V_v(t)} \right) \dot{\omega}_w(t). \]  

(14)

as a general approach for estimating slip (we used simple Euler integration for the implementation of this technique in our simulations). Above we have illustrated one possible method for approximating slip; other investigations have indicated that the FMRLC also works well for other slip estimation methods similar to those described in [2], [3], and [19].

\[ \text{The use of an accelerometer for ABS systems raises issues of noise, road gradient effects, and integration error which need to be more fully investigated in future work.} \]

IV. FUZZY MODEL REFERENCE LEARNING CONTROL FOR ABS

A. FMRLC Design

For the FMRLC-based ABS we use \( \epsilon(kT) = \lambda_v(kT) - \lambda(kT) \) where \( \lambda_v(kT) = 20 \% \text{ (} T = 1 \text{ ms}) \) and \( \epsilon(kT) \) is defined in (2). We utilize a direct fuzzy controller that has 11 fuzzy sets with membership functions uniformly distributed on each (normalized) input universe of discourse. All membership functions used in our FMRLC are triangular shaped with a base-width of 0.4 (except when it is appropriate to use trapezoidal shapes for the outermost regions of the universes of discourse). The triangular membership functions for the fuzzy controller output (normalized) universe of discourse are initially set to be centered at zero indicating that the fuzzy controller initially does not know how to specify the control input (this is what the FMRLC will learn how to do). The normalizing controller gains for the error, change in error, and the controller output are chosen to be \( g_e = 1, g_c = 1/1000, \) and \( g_w = 2200 \).

The reference model for this process was chosen to be

\[ \frac{d\lambda_m(t)}{dt} = -10.0 \lambda_m(t) + 10.0 \lambda_v(t). \]  

(15)

The inputs to the fuzzy inverse model include the error and change in error between the reference model and the wheel slip expressed as

\[ \lambda_v(kT) = \lambda_m(kT) - \epsilon(kT), \]  

(16)

\[ \lambda_v(kT) = \frac{\lambda_u(kT) - \lambda_u(kT - T)}{T}, \]  

(17)

respectively. For these inputs, 11 fuzzy sets are defined with triangular shaped membership functions which are evenly distributed on the appropriate universes of discourse. The normalizing controller gains associated with \( \lambda_v(kT), \lambda_u(kT), \) and \( p_v(kT) \) are chosen to be \( g_{\lambda_v} = 1, g_{\lambda_u} = 1/1000, \) and \( g_p = 2200 \) using the gain selection procedure described above. In a typical braking system, an increase in the braking torque \( T_b(kT) \), will generally result in an increase in the wheel slip. This implies that the incremental relationship between the process inputs and outputs is monotonically increasing. Consequently, the knowledge base array shown in Table I was employed for the fuzzy inverse model. In Table I, \( \Lambda_j^i \) is the \( j \)th fuzzy set associated with the error signal \( \lambda_v \) and \( \Lambda_k \) is the \( k \)th fuzzy set associated with the change in error signal \( \lambda_c \). For convenience, rather than listing the indices \( i \) for \( \Lambda_{j,k}^i \) in the body of the table, we list the center values of triangular membership functions corresponding to the fuzzy inverse model output fuzzy sets \( \Lambda_{j,k}^i \).

B. Performance for Various Road Conditions

The FMRLC described above was simulated for the automotive ABS system. The results of this simulation for wet asphalt and for an icy surface are shown in Figs. 3 and 4, respectively. For these simulation results, only one brake was applied. The braking action was initiated when the vehicle was moving 25 m/s (approximately 56 mph) on a level surface (\( \theta = 0 \)) and we


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ANTI-SKID BRAKING SYSTEM PERFORMANCE ON WET ASPHALT - FMRLC

Fig. 3. Simulation results for FMRLC of a vehicle braking system on a level wet asphalt surface.

Fig. 4. Simulation results for FMRLC of a vehicle braking system on a level icy surface.

Note that for the wet asphalt case, the braking system slip value tracked the reference model output almost perfectly. As a result, the system does not exhibit the limit cycle effect for which many ABS systems are designed (of course, when implemented the algorithm would exhibit some type of cycling to achieve regulation). It is an important future direction to fully investigate the implications of using an ABS system that is not specifically designed to "cycle" in the conventional manner (for example, the effects on steerability). Also note that the braking torque for this case was very smooth. The controller seems to have found the appropriate level of braking torque which needs to be applied to the wheels to maintain a slip of 20%.

Although the simulation results for the icy surface shown in Fig. 4 are likely to be considered acceptable by most control engineers, they are not as good as the results obtained for wet asphalt road conditions. In general, it is very difficult to control slip on an icy surface due to the fact that a very small braking torque is likely to cause lock-up. In fact, to avoid lock-up the controller sets $T_b(t) = 0$ initially, then once the slip goes below the setpoint, the controller applies an appropriate input to regulate the slip to 20%. Notice that the control input increases linearly in Fig. 4 for $t \geq 3$ sec. due to the increasing viscous friction term in (8).
TABLE II
STopping Distance For A Single Wheel ABS System ImpleMented Using FMRLC Versus A Single Wheel Lock-Up Braking System

<table>
<thead>
<tr>
<th>Road Surface</th>
<th>FMRLC</th>
<th>Lock-Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry Asphalt</td>
<td>32.721</td>
<td>38.421</td>
</tr>
<tr>
<td>Wet Asphalt</td>
<td>35.300</td>
<td>39.863</td>
</tr>
<tr>
<td>Ice</td>
<td>151.070</td>
<td>247.257</td>
</tr>
</tbody>
</table>

Table II illustrates the potential of the ABS system described above by comparing the stopping distance which resulted for the FMRLC algorithm with the case where the wheel is locked. Note that a substantial decrease in the stopping distance is obtained on all road surfaces which were considered in Fig. 2 (the plots for dry asphalt were omitted in the interest of space as they were similar in shape to the wet asphalt case).

C. Transitions Between Road Conditions

The next set of simulations illustrates the effectiveness of the FMRLC algorithm for transitions between various road conditions. Here we consider two very likely real world scenarios. The first involves the situation where the brakes are applied on wet asphalt and during the braking action the vehicle moves onto an icy surface. Notice that during the initial braking action, the wet asphalt would allow for a relatively large braking torque without lock-up occurring. However, when the vehicle reaches the icy road condition braking torque must be reduced quickly to prevent lockup. This large system variation requires a very demanding controller modification on the FMRLC algorithm. However, the simulation results for this scenario shown in Fig. 5 illustrate that the FMRLC algorithm is capable of dealing with such drastic process variations.

The second case involves the reverse of the situation described above. In this case, the brakes are applied on an icy surface and during the braking action the vehicle moves onto wet asphalt. This situation would require the FMRLC to reconfigure itself to increase the torque when the vehicle reaches the wet asphalt. Fig. 6 illustrates the simulation result for this scenario. Once again the FMRLC was successful in learning to compensate for the adverse road conditions.

V. Conclusions

The principal objective of this paper was to illustrate the design methodology and application of the new FMRLC algorithm for an automotive antiskid braking system which is subjected to harsh road conditions. While the behavior of a conventional braking system varies significantly for different road and operating conditions, the results obtained in this paper (although somewhat preliminary) indicate that the FM-
RLC provides a promising approach to maintaining effective braking even under adverse road conditions. Directions for future research include: 1) testing the FMRLC design for other harsh road conditions (e.g., snowy roads, transitions in road conditions involving snow); 2) fully comparing the approach to conventional control algorithms for ABS (such as a gain-scheduled PD controller); 3) testing the approach with a full nonlinear dynamical vehicle simulation; and 4) studying implementation characteristics of the FMRLC-based ABS system.

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